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EXERCITATIO GEOMETRICA DE DESCRIPTIONE *Linearum Curvarum.*

Auctore *GULIELMO BRACKENRIDGE*,
Eccleſiæ Anglicanæ Presbytero.

LONDINI:
Apud Ric. Hett & Joh. Nourse Bibliopolas.
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ETSI Vir honoratiſſime non ſatis mihi conſtiterat de tua virtute, amore Literarum, & progreſſu quem in Matheſeos ſtudio feciſti, quibus adducerer ut Lucubratiunculas haſce tuo nomini inſcriberem; mira tamen tua benevolentia in me, conatuſque meos, effecit ut nullus erat cui potiori jure, hanc teſſeram gratitudinis meæ publice tribuam. Merito dignitas tua me cohiberet, niſi ſatis compertum habui, te omni etiam minimo delectatum quod Geometriam ſapit, aut utile eſt in humanioribus Literis. Nemo enim qui tua conſuetudine utitur neſcit quantam operam in ſcientiarum ſtudio poſuiſti, quæ plurimum perpoliunt ingenium, & aptum reddunt ad magna negotia rei-publicæ ſuſcipienda; cu-

jus

jus tutelam ab inclytis majoribus tibi mandatam habes. Et exemplar quidem perilluſtre ab illis accepiſti; nam omnibus notum, qui rerum Scoticarum vel leviter gnari, quam ſtrenui publicæ Libertatis vindices fuere, & in ſumno rerum diſcrimine quantum conciliis & armis profuere. Sed de magnis præſtat ſilere quam levia dicere. Vereor autem ne in modeſtiam tuam peccem ſi dicerem, Patriam tuam non minora a te expectare, quam a clariſſimo quovis e Proavis tuis. Honoris enim & Libertatis principiis, quæ generoſos ad magna capeſſenda propellunt, te imbutum non tantum ſperandum eſt, ſed porro cuivis exploratum. Cæterum quis non videt, Philoſophum & Mathematicum, magnis Regni negotiis occupatum præ reliquis ſuæ ætatis, qui puerilia tantum meditantur, longe in faſtigium gloriae progreſſurum. Perge illuſtriſſime vir, virtutem & doctrinam ſequi, ut ſis tandem non tantum Britaniae, ſed & ſeculi decus egregium. Interim ignoſcas mihi quaeſo meam audaciam, quod patrocinium tuum, huic opuſculo invocaverim. Et ut omnia felicitur ſuccedant quae digne conaris, Deum Opt. Max. Precatur

Tui Nominis & Dignitatis
Obſervantiſſimus Cultor

GUL. BRAIKENRIDGE.

PRÆ-

PRÆFATIO.

CUM in hoc tandem ſeculo, Geometria multos habeat cultores, & quaſi ad culmen evecta videatur, ejuſque auſpicio plures alantur artes, ut nemo Naturæ ſtudioſus non fatebitur; viſum eſt in hac exercitatione Theoremata quædam proponere, quæ mathematicis haud ingrata fore, nec prorſus inutilia, libentur ſpero. Nec eo minus placitura confido, quod ſint de Geometria abſtracta & Curvarum natura; quid enim hodiernis hactenus jucundius, aut uberiores protulit fructus quam Curvarum contemplatio; ſive uſum ſpectes in dificilioribus Problematis ſolvendis, aut voluptatem qua replent animum earum harmoniam & proprietates perluſtrando.

Innumera ſunt naturalis Philoſophiæ Problemata quæ ſine adminiculo Curvarum etiam altiorum generum, nec ſolvenda ſunt, nec demonſtranda. Hujuſmodi adhibet plurima *illuſtriſſimus Neutonus*, quæ ſibi & aliis inexplicanda fuiſſent, niſi Curvarum proprietates penitus intellexiſſet. Nemo autem facile dixerit Philoſophiam

phiam naturalem inutilem eſſe: fruſtra tamen naturam perſequeris niſi primum Matheſin puram intelligis. Philoſophia & naturæ Hiſtoria longe inter ſe differunt, Phœnomena enim in genere, & eorem ſeriem multis obſervationibus confirmatam capere potes; ſed de cauſis vere & accurate differere, ſine Geometriæ etiam reconditioris auxilio eſt omnino impoſſibile & argumentum ignorantiæ procul dubio proferres. Ita olim ſentiebat *Cl. Verulamius* ſui ſeculi perenne decus. " Multæ quidem inquit naturæ partes, nec ſatis ſubtiliter comprehendi, nec ſatis perſpicue demonſtrari, nec ſatis dextre & certo ad uſum accommodari poſſunt, ſine ope & interventu Mathematicæ. Prout enim Phyſica majora indies incrementa capiet, & nova axiomata educet, eo Mathematicæ opera nova in multis indigebit, & plures demum fient Mathematicæ mixtæ. De Aug. Scien. Lib. iii. Cap. 6.

Veteres Geometriam intra anguſtos limites concluſerunt, & nihil unquam de Curvarum univerſali natura cogitavere, animos tantum ad *Sectiones Conicas*, *Ciſſoidem*, *Conchoidem*, & quaſdam alias Mechanicas Lineas, *viz. Spiralem*, & *Quadraticem*, applicavere. Nec fas erat Geometris per plurima ſecula ulterius progredi. Recentiores vero tandem libertatis vindices vetuſtos fines perrupere, & Lineas Curvas univerſaliter tractare cœperunt, eaſque in ordines diſtinguere, pro varia relatione ad rectas ex æquatione Algebraica cognita. Atque inde Matheſis incredibile incrementum habuit, quod & grato anino agnoſcere Celeberrimo auctori *D. Carteſio* par eſt. Veruntamen magnopere deſi-

deſiderabatur ad hanc Theoriam perficiendam methodus generalis & facilis, qua Curvæ potuerint in plano deſcribi, ut earum ope conſtruerentur problemata: & hinc dedita opera varie ſeſe exercebant *Geometræ* quarundam Curvarum deſcriptiones ſatis faciles inveniendo. Primus autem omnium generalem methodum monſtravit *acutiſſimus Neutonus*, ope motus angulorum circa duo puncta moventium, quod & eo magis mirandum quia dabantur tunc nulla principia ex quibus conſequi potuit talis Theoria. Hæc diu meditatus in aliam a Neutoniana longe diverſam, & ni fallor ſimpliciorem incidi, quam hic explicatam habes, & quoad ſcio novam & inauditam reperies, Sed quantum valeat, aliorum eſto judicium.

In concinnandis demonſtrationibus algebraico calculo plerumque utor, ita enim Theoremata ipſe inveni, & nunc piget more veterum Geometricas demonſtrationes componendo tempus terere. Hoc officium quibus otium eſt non invitus relinquo. Cognita prima Prop. cætera ſponte fluunt, nec ſunt inventu difficila, & propterea prolixius egi in Corollariis Prop. 1. recitandis quæ lucem adferunt ſequentibus. Sed ne opus nimium creſceret Theoremata de locis Geom. *illuſtriſſimi March. de l'Hoſpital* tanquam notiſſima ſuppono. Præterea hoc Theorema, *viz. duæ Lineæ cujuſcunque ordinis iu tot punctis ſeſe interſecare poſſunt, quot ſunt unitates in rectangulo ſub earum indicibus*, tanquam poſtulatum aſſumo ut & alii qui in ejus demonſtratione hærere videntur. Cl. *Geometra D. Georgius Campbel* inter plurima alia eximia inventa, quæ ob ſummam ejus modeſtiam apud eum latent, hujus theorematis præclaram habet demonſtrationem,

tionem, in lucem brevi ut ſpero prodituram. Fuſe quidem & laxe rem totam tractavi, ſed & hoc conſulto feci, ut tyronibus aliquatenus ſtudeam; quod utinam & alii feciſſent, nam plurima Mathematica ſcripta ſunt adeo involuta & conciſa, quaſi auctores artem celare voluiſſent, ut multos deterrent ab eorum lectione, quibus nec otium, nec patientia, nodos & involucra explicare.

Anno 1726 cum *Edinburgi* degebam, Propoſitiones. 1, 4, 5, 21, 22, 23, inveni, quod & tunc indicavi doctiſſimo D. *Georgio Martin*, M. D. ad *Fanum Andreae*. Et inſequenti anno 1727, per tres menſes commoratus *Londini*, eas impertivi celeb viro D. J. *Craig* Matheſeos peritiſſimo, qui me hortatus eſt ut publici juris facerem. Poſt paucos dies accidit ut Cl. D. *Maclaurin Edin. Prof.* viſerem qui tunc Londini fuit, & ille occaſionem nactus retulit ſe colloquium habuiſſe cum D. *Craigio*, qui Theoremata mea ſibi narravit; dixitque inſuper ſe quædam inveniſſe ſimilia, & MSS oſtendit quo innuebat inventa ſua contineri; ſed qua ratione ductus neſcio in manus meas non tradidit nec licuit in illud vel leviter inſpicere. Poſtea vero in *Scotiam* rediturus chartulam in qua erant Prop. predictæ depoſui apud D. *Georgium Gordon*; ut communicaret cum amico ſuo Cl. D. *Deſaguliers Philoſophiae Experimentalis Profeſſore*, & hic demum cum *Regia Societate* ſed infortunio quodam, ut mihi dixit, intercidit chartula.

Denique benevole Lector, ſi hæc tibi non movent faſtidium, plura & forſan majora, poſt paucos menſes in una aut altera exercitatione videbis; interim vero utere & vale.

EXER-

EXERCITATIO GEOMETRICA DE DESCRIPTIONE LINEARUM CURVARUM.

SECTIO I.

De Deſcriptione Curvarum Primi Generis ſeu Linearum Ordinis Secundi.

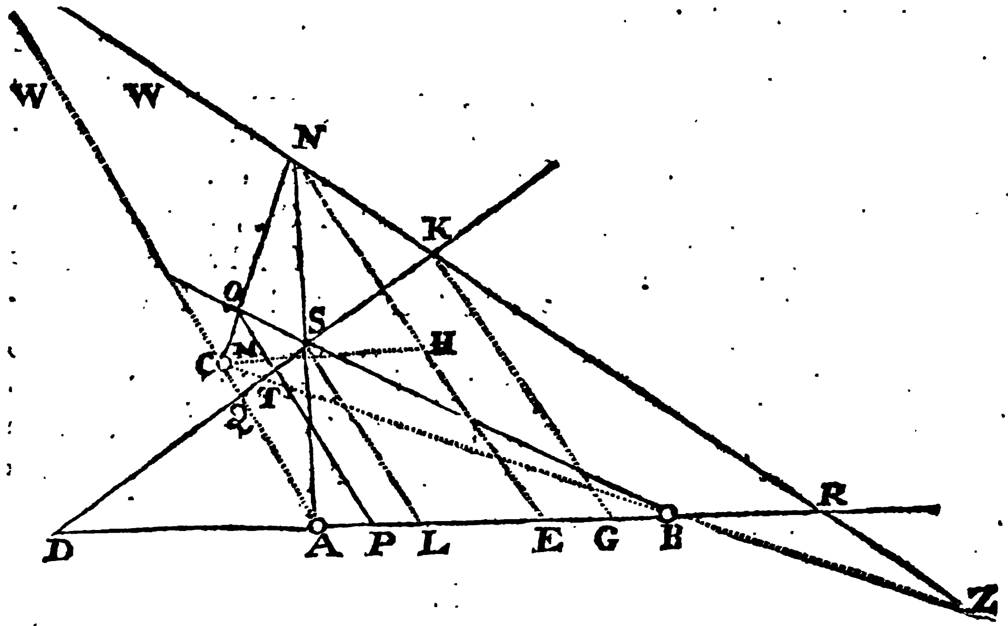
PROP. I.

CIRCA tria puncta data A, B, C, in plano quovis tanquam polos, moveantur tres rectæ ASN, BSO, CON, quæ ſeſe interſecent in punctis S, N, O, & ducantur duo interſectionum puncta S & N per rectas DSK, RKN poſitione datas; dico reliquum O Sectionem Conicam deſcribere.

Per puncta data A, B, C agantur duæ rectæ DABR & ACQ ſibi occurrentes in A, quarum prior DABR datis poſitione DK, KR, occurrat in D & R, altera vero CA rectas DK & RK, ſecet in Q & W, conveniant vero poſitione datæ DSK, RKN in K; atque ex punctis O, K, S, N, ducantur rectæ CA, parallelæ OP, KG, SL, NE, quæ occurrant rectæ DABR in P, G, L, E; & a puncto dato C agatur CMH rectæ DABR parallela, rectas OP, NE, ſecans in M & H: ideoque datæ ſunt rectæ AB, AR, AD, KG, CA = MP = HE.

$= HE$. Ponatur $AB = a$, $AR = b$, $AD = d$, $CA = c$, $AE = u$, $AL = z$, curvæ ordinata $OP = y$, abſciſſa $AP = x$; & quia dantur latera triangulorum KGR, DKG, ſit $GR : GK :: a : g$ & $DG : GK :: a : k$; & propterea ob triangula KGR, DKG, triangulis NER, DLS, ſimilia erit $a : g :: RE : NE$, & $a : k :: DL : LS$: & inde $NE = \frac{gb - ug}{a}$, & $LS = \frac{dk + zk}{a}$. Quoniam igitur triangula AEN, ASL, ſunt æquiangula, & $AE : AL :: EN : SL$, erit $u : z :: \frac{gb - ug}{a} : \frac{dk + zk}{a}$ & ideo $u \times \overline{dk + zk} = z \times \overline{gb - ug}$, & $z = \frac{ukd}{gb - gu - ku}$. Cumque triangula BPO, BSL ſunt etiam ſimilia, & $BP : BL :: PO : SL$: erit $a - x : a - z :: y : \frac{dk + zk}{a}$, & proinde $a^2y - azy = dk \times \overline{a - x} + kz \times \overline{a - x}$, & $z = \frac{a^2y - dka + dkx}{ay + ka - kx}$: unde $\frac{udk}{gb - gu - ku} = \frac{a^2y - dka + dkx}{ay + ka - kx}$, & reducendo evadet $u = \frac{gba^2y - gbakd + gbxdk}{aydk + ga^2y - gadk + gxdk + ka^2y}$. Cæterum propter triangula CMO & CHN æquiangula, eſt $CM : MO :: CH : HN$; quare ob parallelas CH & AB erit $AP : OP - CA :: AE : NE - CA$, adeoque $x : y - c :: u : \frac{gb - ug - ca}{a}$, atque ita $yua - cua = xbg - xgu - xac$: & invenitur $u = \frac{xbg - xac}{ya - ca + xg}$. Æquentur duo valores reperti rectæ $AE = u$, & ducendo numeratorem unius in denominatorem alterius prodibit Æquatio Curvæ ad abſciſſam AP deſcriptæ, $ga^2by^2 + \left.\begin{matrix}\frac{dc - bg}{g} \times ka \\ k \times a^2c\end{matrix}\right\} \times yx - gba \times \overline{ac + kd} \times y + cgdkx^2 - kdg \times \overline{bc + ac} \times x + gbakdc = 0$. Ergo cum ordinata y, & abſciſſa

abſciſſa x ad duas tantum dimenſiones aſcendant, patet figuram motu puncti O deſcriptam eſſe Sectionem Conicam.



COROL. I.

Curva habebit quinque puncta data K, B, R, C, & Q. 1mo. Interſectio K rectarum RKN & DKS eſt Curvæ punctum, quia coincidunt ibi puncta S, N, & O. 2° Curva tranſibit per B & R, cum enim punctum deſcribens O decuſſet rectam DAR evaneſcet ordinata OP $= y$, & propterea Æquatio evadet $x^2 \begin{smallmatrix}-a\\-b\end{smallmatrix} x + ab = 0$, cujus radices ſunt $+a$ & $+b$, vel $+$BA & $+$AR. 3° Denique Curva attinget puncta C & Q, nam ſi ordinata OP $= y$ tranſeat per Polum A, ibi coincidet cum recta ACQ, atque evaneſcet abſciſſa AP $= x$ & ideo proveniet Æquatio $y^2 + \frac{ca + kd}{a} x - y + \frac{kdc}{a} = 0$, cujus, radi-

radices erunt $+c$ & $+\frac{kd}{a}$, ſeu $+$AC & $+$AQ; etenim AQ$=\frac{kd}{a}$, quia $a : k :: $ AD : AQ. Hæc etiam ſine calculo eliciuntur. Per polos C & B agatur recta CB, quæ rectis KD & KR occurrat in T & Z. Manifeſtum eſt quando interſectio S rectarum ASN, BSO, per veniat ad T, coincidet recta BSO cum BC, & punctum O Curvam deſcribens erit locatum ad C; atque ſimiliter cum interſectio N rectarum ASN, CON, attinget Z, recta CON cadet in CB, & punctum O tranſibit per B. Præterea quando S veniat ad D ſimul accedent rectæ ASN & BSO ad BAD, & puncta N, & O coibunt ad R. Si vero punctum N movendo per rectam NKR attingat W, coincident rectæ CON, ASN, cum ACQ, atque puncta S & O coibunt ad Q. Notandum tamen ſi concurſus N rectarum CON & ASN duceretur, per rectam DK, atque concurſus S rectarum ASN & BSO per rectam KR, Curva tranſiret per puncta D & W.

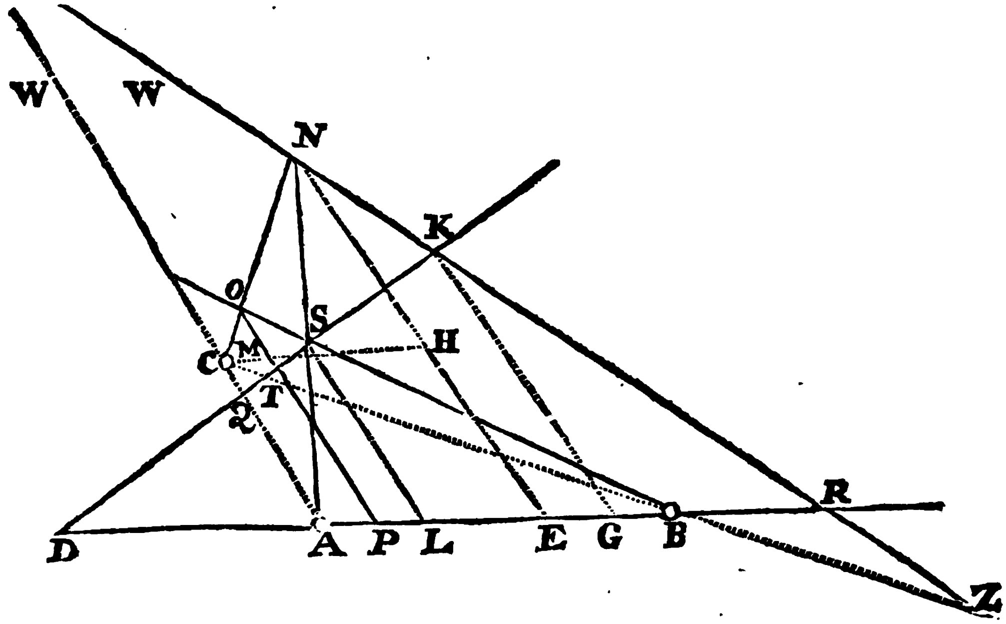
COROL. II.

Si alterutra rectarum KD & KR tranſeat per polum A, Punctum O in recta movebitur. Ponamus enim rectam KR duci per Polum A, & evaneſcent rectæ AR & NE & ipſa abſciſſa AP $= x$, coincidentque puncta AR, NE, atque recta a polo C ducta ſemper tranſibit per A. Jungantur puncta C & A atque recta AC erit locus puncti O. Tranſeat autem altera DK per polum A, atque ideo evaneſcent rectæ AD, SL, & ipſa ordinata OP $= y$, coincidentque puncta AD, SL, & recta a polo B ducta continuo tranſibit per A. Agatur recta per B, A, & hæc erit ipſa linea motu puncti O deſcripta.

COROL. III.

Incidat recta KR, in qua movetur concurſus N rectarum CON, ASN, in polum C, aut etiam altera DK, in qua movetur concurſus S recta-

rectarum ASN, BSO in polum B, & punctum O eaſdem rectas RK & DK, percurret.

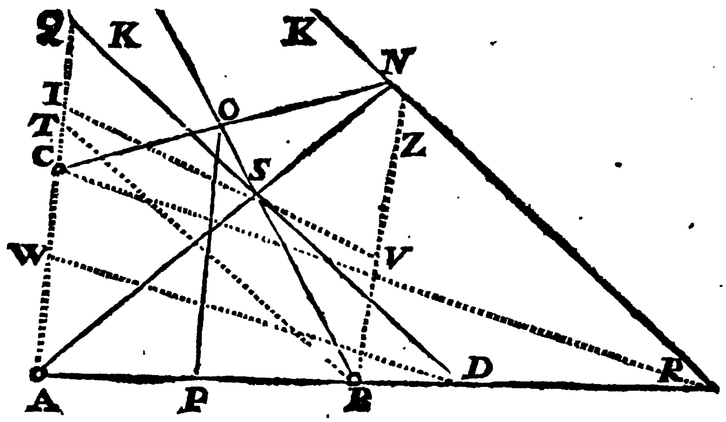


COROL. IV.

Si polus C ſit in recta KD, & interea concurſus N rectarum CON, ASN percurrat rectam NKR, aut etiam polus B accedat ad rectam RK dum concurſus S rectarum BSO, ASN, moveatur per rectam DK; Figura deſcripta rectas ACW & DAB tanget in C & B. Nam in primo caſu erit $d : c :: a : k$, & inde $c = \frac{kd}{a}$, atque adeo cum ordinata PO tranſeat per polum A, & evaneſcat $AP = x$, provenit Æquatio $y^2 + \frac{ca + kd}{a} \times -y + \frac{kdc}{a} = o$, cujus radices $+c$ & $+\frac{dk}{a}$ erunt æquales. At in altero caſu erit $a = b$, & ideo cum evaneſcat ordinata $PO = y$, emerget $x^2 + \overline{a + b} \times -x + ab = o$, cujus radices $+a$ & $+b$ ſient æquales. Unde liquet rectas ACW & DAB Figuram tangere in punctis C & B.

COROL.

COROL. V.



Quod ſi rectæ DK & RK poſitione datæ ſint parallelæ, & rectam AB ſecent in D, R; Curva evadet Hyperbola. Nam ſi puncta D, R, ſint ad eaſdem partes ipſius A, quia angulus KDA æqualis eſt angulo KRA, & ideo $k = g$, orietur Æquatio $y^2 + \frac{cd + gb}{ab} xy + \frac{gd + ca}{a} x - y + \frac{cgd}{a^2 b} x^2 + \overline{a + b} \times - \frac{cgdx}{a^2 b} + \frac{cgd}{a} = 0$. Capiatur dimidium ex coefficiente termini xy, nempe $\frac{cd + gb}{2ab}$, & ejus quadratum $\frac{c^2 d^2 + 2cdgb + g^2 b^2}{4a^2 b^2}$, majus erit coefficiente $\frac{cgd}{a^2 b} = \frac{4bcdg}{4a^2 b^2}$ Termini x^2; eſt enim $c^2 d^2 + g^2 b^2$ major quam $2cdgb$ (excepto tantum caſu corolarii ſequentis cum $cd = gb$) & addito utrinque $2cdgb$, fiet etiam $c^2 d^2 + 2cdgb + g^2 b^2$ major quam $4bcdg$: adeoque per Theorema notiſſimum de Locis Geometricis Figura eſt Hyperbola. Jungantur puncta C, R, & per B, D, ducantur rectæ BT, DW rectis DS, CR, parallelæ, ſecentque rectam CA in T, W, & a polo B ducta BF rectæ CA parallela, in ea capiatur

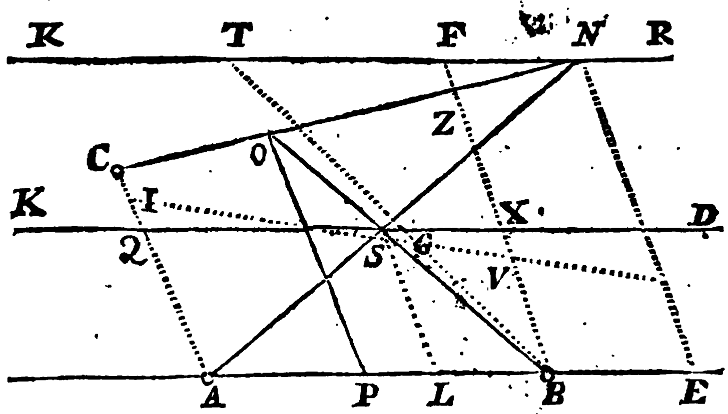
atur $BZ = AQ + TC - AW$; & Z erit Curvæ punctum. Biſſecentur rectæ CQ & BZ in I, & V & acta IV, hæc erit Diameter ad Ordinatas IQ, VZ. Si vero puncta D & R ſint ad contrarias partes puncti A, emerget æquatio $y^2 + \frac{bg - dc}{ab} \times xy - \frac{dgc}{ab} \times x^2 + \frac{ac - gd}{a} \times y - dgc \times \frac{\overline{b+a} \times x}{a^2 b}$ $- \frac{gdc}{a} = 0$, & cum quadratum ex dimidio coefficientis Termini xy, nempe $\frac{b^2g^2 - 2bgdc + d^2c^2}{4a^2b^2}$ majus eſt coefficiente $\frac{-dgc}{a^2b} = \frac{-4bgdc}{4a^2b^2}$ Termini x^2; Figura erit etiam Hyperbola.

COROL. VI.

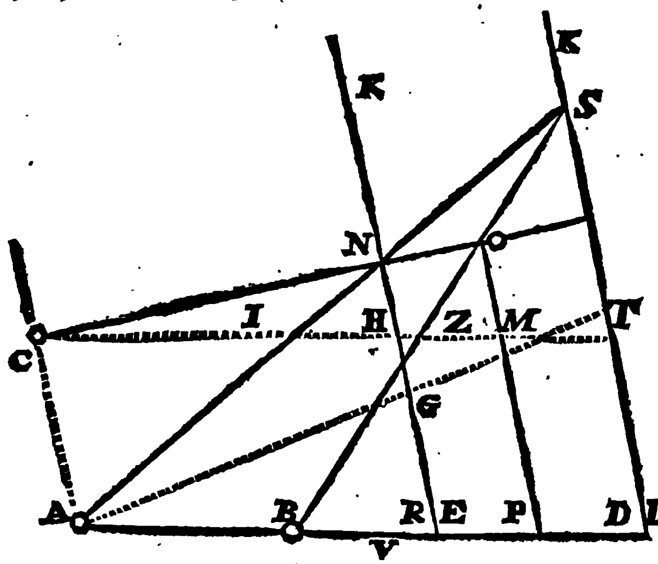
Cæteris manentibus ut in Corol. præced. ſi D & R ſint ad eaſdem partes ipſius A, & punctum T ita locetur in recta CA ut ſit $AR : AC :: AD : AT$. Figura mutabitur in Parabolam. Nam quia $AR \times AT = AC \times AD$, erit quoque $bg = cd$, & proinde $c^2d^2 + 2cdbg + b^2g^2 = 4cdbg$, & ideo quadratum ex dimidio coefficientis Termini xy æquale erit coefficienti Termini x^2. Igitur per Theorema jam citatum Figura eſt Parabola. Sumatur $BZ = QA + CA - 2AT$, & biſſectis CQ & BZ in I & V, ducatur recta IV; hæc erit Figuræ Diameter ad ordinatas QI, ZV.

COROL. VII.

Si rectæ KSD, KNR, ſint alterutri rectarum AB, AC parallelæ, Figura erit Hyperbola. Sint enim NR & DS, rectæ AB parallelæ, & proinde datæ erunt NE & SL. Ponatur $EN = g$, $SL = k$, & provenit Æquatio Curvæ $xy + \frac{agy^2}{ck} - \frac{ga \times \overline{k+c}}{kc} xy - gx + ga = 0$, in qua non invenitur Terminus x^2 quod indicio eſt Hyperbolam deſcribi, & ejus



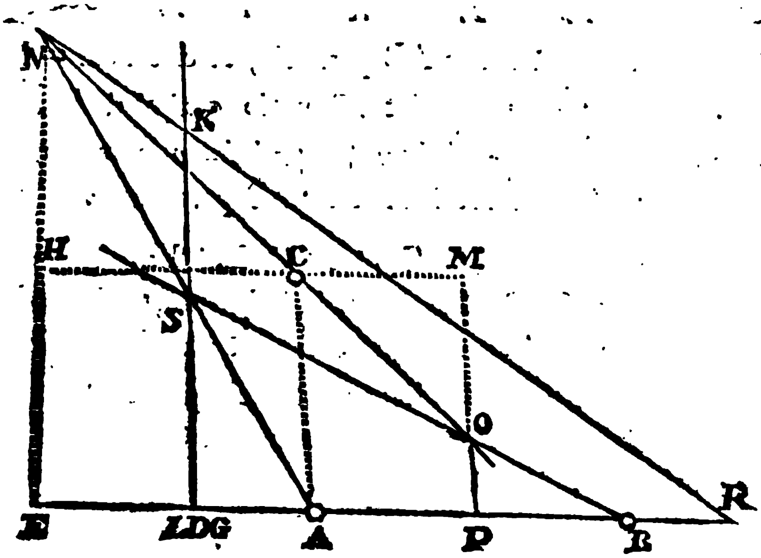
ejus aſſymtoton rectæ AB parallelam. Ducatur BF a polo B rectæ AC parellela, & rectas NR, QD ſecans in F, X, dein in recta RK, capiatur FT = CA, et ducta BT, quæ rectæ QD occurrat in G, & in recta BE, fiat BZ = SL + CA - GX; & z erit punctum Curvæ: biſſecentur rectæ CQ, BZ in I, V, & acta IV, ea erit Curvæ, Diameter. Recta vero



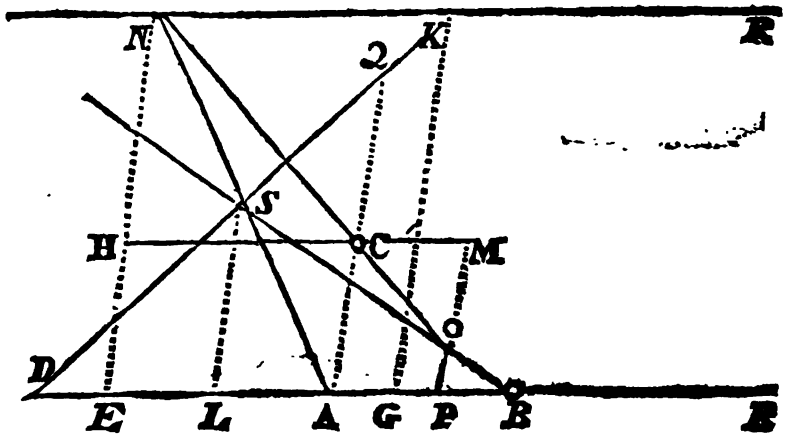
RNK eſt Aſſymptotos. Et ſimiliter ſi DSK, RNK ſint rectæ CA parallelæ, coincident puncta R, E & D, L, & habebitur Æquatio ejuſdem

dem formæ cum precedente $xy + \frac{dcx^2}{ab} - dy - \frac{cd \times \overline{b+a} \times x}{ab} + dc = 0$, adeoque figura eſt Hyperbola, & recta DS, ejus aſymptotos. A puncto D, in recta DS, cape DT = CA, & ducatur AT, rectam RK ſecans in G, atque in CM rectæ AB parallela, ſumatur CZ = AB + AR − RG, dein biſſecenter rectæ CZ, BR in I, V, & ducta IV, hæc erit Diameter.

COROL. VIII.



Quod ſi unum e duobus punctis Q, R, (quæ per Corol. 1. ſunt Curvæ puncta) abeat in infinitum, ſeu quod eodem redit ſi DK ſit rectæ AC, vel KR ipſi AB parallela; figura erit etiam Hyperbola. Ponamus enim DK rectæ AC parallelam, & coibunt puncta L, D, G, ac obtinebitur Æquatio $cdgx^2 + \overline{db + ca - gb} \times y \times a + \overline{b+a}x - dgcx - adgby + adgbc = 0$: unde patet figuram aſymptoton habere rectis DK, AC, parallelam.



Detur autem KR rectæ AB parallela, & erit NE = KG, & ideo ſi pro NE ſcribatur g, proveniet Æquatio $ga^2y^2 + k\times\overline{dc+ac-ag}\times xy - ga\times\overline{kd+ca}\times y - ckdgx + cgakd = 0$, quæ pariter innuit Curvam aſymptoton habere rectis KR & AB parallelam.

COROL. IX.

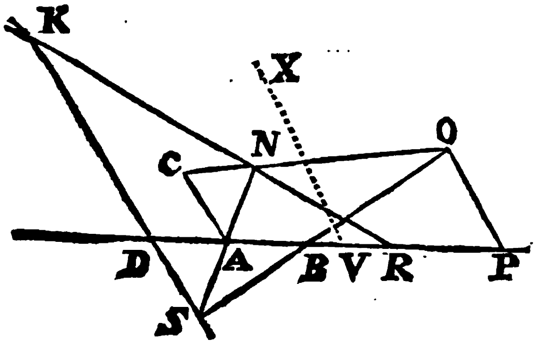
Abeant puncta Q & R ſimul in infinitum, vel quod eodem redit ſint DK, KR rectis AC, AB, parallelæ, & ideo convenient, D, L, G, fietque KG=NE=g; & inde orietur Æquatio $+\overline{dc+ac-ag}\times yx - cdgx - adgy + cadg = 0$: & propterea Curva duas Aſymptotos habebit rectis AB, & AC parallelas.

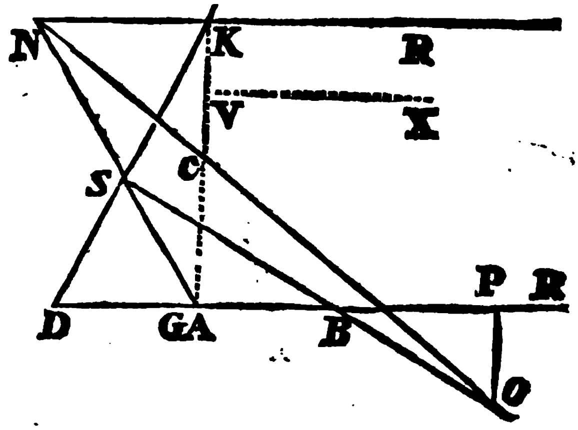
COROL. X.

Stantibus quæ in Corol. 8. fiat in primo caſu $d+a=\frac{bg}{c}$, & in altero $d+a=\frac{ag}{e}$, & ideo terminus xy evaneſcet, quod indicio eſt Parabolam deſcribi. Ponatur autem in primo caſu DK=DR, & AC=AD=AB=$\frac{AR}{2}$, & Æquatio emerget $x^2 - 3ax - 2ay + 2a^2 = 0$, dein biſſecetur BR in V, & agatur VX rectis DK, AC, æquidiſtans: atque hæc erit Diameter

ad

ad ordinatas rectæ AB parallelas ac ajus Latus rectum erit 2AB & Vertex elongabitur a recta ABR diſtantia $\frac{AB}{8}$. Atſi in altero caſu ſit $GK = 2AC$, & $AC = AB = DA = DG$, coincident puncta G, A, & proveniet $y^2 - 3ay - 2ax + 2a^2 = 0$. In recta KG capiatur



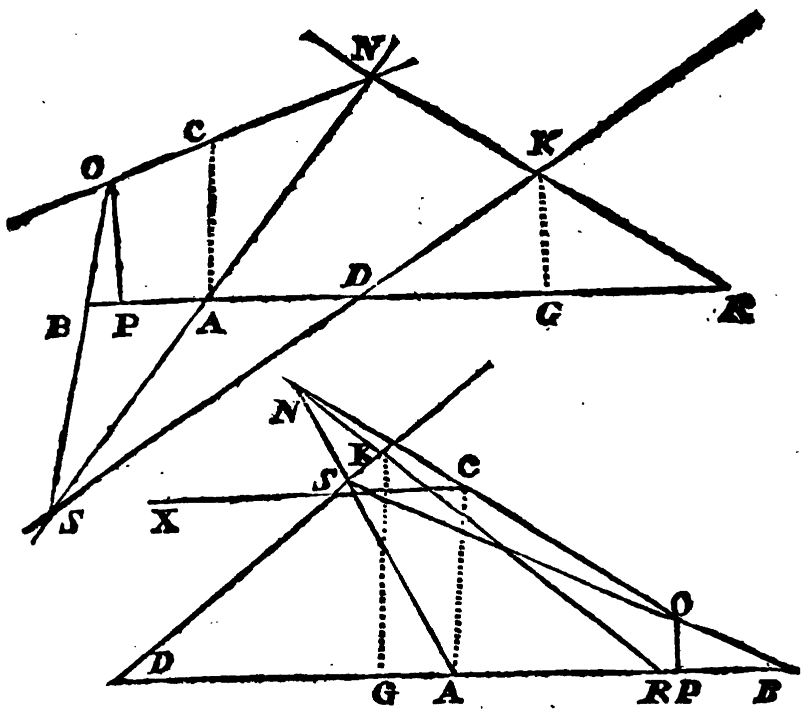


$KV = \frac{AD}{2}$, & acta VX rectis AB, KR æquidiſtante, hæc erit Diameter ad ordinatas rectæ AC parallelas, & Latus rectum fiet 2AB atque Vertex diſtabit a recta ACK longitudine $\frac{AB}{8}$. Unde deducitur facillima Methodus figuram deſcribendi ad Diametrum & datam ordinatam.

COROL. XI.

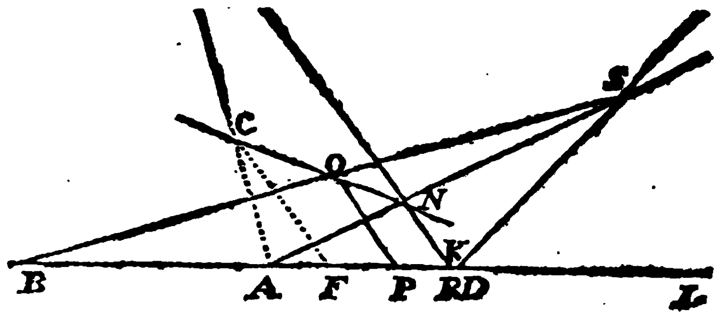
* Sit polus A inter B & D, atque R ad contrarias partes ipſius D, & capiatur $AC = AD$ & $DG = GK = GR$, fiatque $AB : AD :: BD : DR$, & ideo evaneſcent Termini xy & y; & proinde emerget $y^2 + \frac{d^2x^2}{ab} + \frac{d^2 \times \overline{b-a} \times x}{ab} - d^2 = 0$, ubi Terminus x^2 eſt affirmativus: unde colligitur figuram eſſe Ellipſin rectamque BR ejus Diametrum, quæ eſt ad ſuum Latus rectum ut ab ad d^2. Locetur autem polus

* Vid. fig. ſeq. pag.



lus A inter D& R, ita ut B & R ſint ad eaſdem partes ipſius A, ſitque CA=DA & GK=DG=GR, & inſuper AD : AR : : AB : AD+2AB, atque adeo evaneſcet Terminus xy; & calculo facto erit $y^2 - 2dy + \frac{d^2x^2}{ab} - \frac{d^2 \times \overline{b+a}}{ab} \times x + d^2 = 0$, & propterea deſcribetur Ellipſis. Per polum C agatur CX rectæ DAB, æquidiſtans, atque hæc fiet Diameter ad ordinatas rectæ AC parallelas, quæque erit ad Latus rectum ut ab ad d^2. Denique ſunto D & R inter polos A & B, ita ut D ſit proximum polo A. Sumatur CA=DA, & GK=DG=GR, atque AB : AD : : BD : DR, & proinde evaneſcent Termini xy & y, prodeunte $y^2 - \frac{d^2x^2}{ab} - \frac{d^2 \times \overline{b+a} \times x}{ab} - d^2 = 0$. Itaque quia Terminus x^2 eſt negativus, Figura erit Hyperbola Diametrum habens BR quæ eſt ad Latus rectum ab ad d^2. COROL.

COROL. XII.



Accedat interſectio K rectarum KSD & KNR ad rectam BA, & ideo coincident puncta D, R & K, ac figura rectam RKN continget in R: Quinetiam ſi interſectio K ſit in recta CA coibunt puncta Q, W, K *, & figura rectam DKS tanget in Q. Nam ductis ordinata OP & rectis CF & SL rectæ NKR parallelis, rectamque BA ſi opus eſt productam ſecantibus in P, F, & L, ſit RF = f & RL : SL :: a : k, cæteris vero manentibus ut in Prop. prodibit obvio calculo $fa^2y^2 +$

$$\overline{\underset{+ca^2}{f-a-b}} \times bk \times yx + \overline{\underset{-bca^2}{b-f.+a}} \times kb^2 \times y + ckbx^2 + \overline{a-b} \times ckbx - b^2cak = 0$$

adeoque cum abſciſſa AP fuerit rectæ AR æqualis, Termini ſe mutuo deſtruent manente tantum $fa^2y^2 = 0$ ſive $y = 0$. Unde concludi licet figuram in nullo alio puncto rectæ RKN occurrere poſſe, & quidem ipſam RKN contingere in R. Et ſimiliter ratiocinari licet de altera recta DKS, cum interſectio K ſit in recta CA.

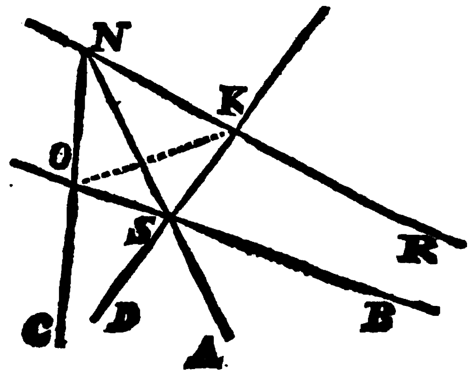
COROL. XIII.

Quod ſi unus aut duo e polis A, B, C, (vid. fig. 1.) locentur ad diſtantiam infinitam; rectæ circa polos ita movebuntur ut ſint ſibi ſemper parallelæ, & datos angulos cum rectis poſitione datis RNK, DSK continuo conficient; Punctumque O deſcribet Sectionem Conicam, niſi unum aut alterum caſum excipias quibus rectam percurret.

COROL.

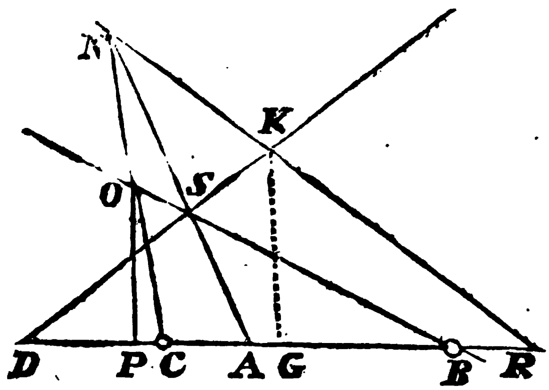
* Vid. Fig. 1.

COROL. XIV.



Abeant vero tres poli A, B, C, in infinitum, & rectæ circa polos movebuntur ſibi ſemper parallelæ, & Punctum O lineam rectam deſcribet. Nam quia recta A S N movetur ſibi parallela, datos angulos A N K, N S K cum poſitione datis RN, DK conficiet, & ideo dabitur ratio rectæ S K ad S N; & quia etiam rectæ B S O, C O N datos angulos K S O, O N K cum rectis D K, RK poſitione datis conſtituunt, dabuntur omnes anguli trianguli S O N, & proinde data erit ratio rectæ S N ad S O, igitur ex æquo dabitur ratio rectæ S K ad S O, atque punctum O continuo rectam O K poſitione datam ductamque per K, continget.

COROL. XV.



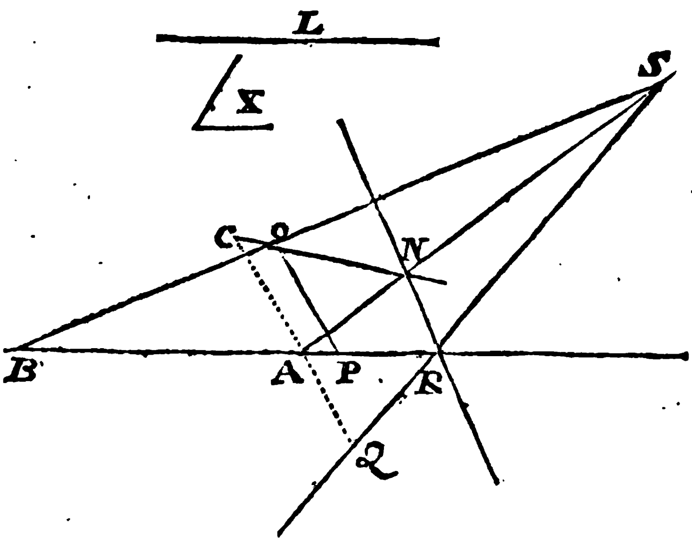
Locentur poli A, B, C in eadem recta, atque punctum O lineam rectam percurret. Ponatur enim DG=GR & DG : GK : : a : g, ſitque ro-

ordinata OP rectæ KG parallela, & iiſdem ſymbolis utendo ut in Prop. prodibit Æquatio. $\overline{2a^2c+ba^2+adc}\times y+\overline{gba-gdc}\times x-gdc\times\overline{b+a}-\overline{gba}\times\overline{c+d}=o.$ Ergo cum ordinata y & abſciſſa x ad unicam dimenſionem aſcendant; patet punctum O rectam deſcribere.

In hac Prop. & ſequentibus; polos circa quos moventur duæ rectæ, quarum interſectione mutua deſcribitur Curva, & per quos etiam ipſa ſemper tranſibit, deſigno literis B & C; Litera vero A, reliquum polum, circa quem movetur recta, cujus duæ interſectiones cum duabus prædictis per rectas poſitione datas ducuntur, & per quem ipſa Sectio Conica deſcripta nunquam tranſibit. Quod & eò diligentius obſervandum, ut vitetur confuſio, & facilius ratiocinia noſtra capias. Vid. Corol. 1

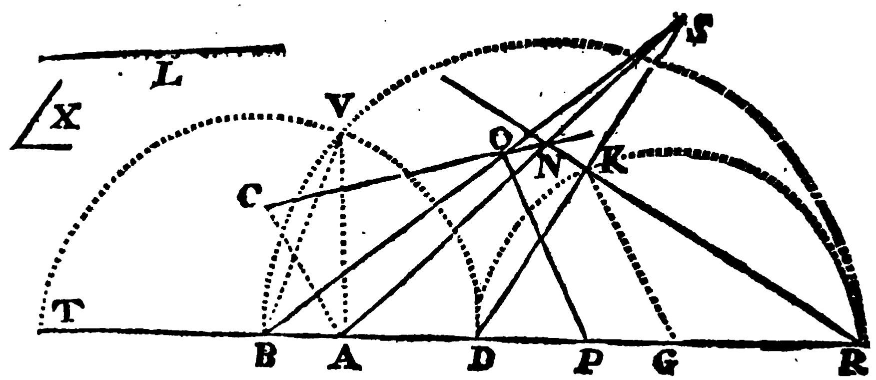
PROP. II. PROBLEMA.

Datis Diametro BR & angulo X, quem cum ea faciant ordinatæ, & Latere recto L, Sectionem Conicam deſcribere.



Pro Ellipſi. Secetur BR in A ut ſit BA : AR : : BR : L, & ad punctum A conſtruatur angulus BAC, æqualis dato X, atque producta CA.

CA utrinque cape CA & AQ æquales rectæ AR. Per punctum R agatur RN rectæ AC parallela, & jungantur R & Q; dein circa puncta A, B, C, tanquam polos moveantur rectæ ANS, BOS, CON ſeſe interſecantes in punctis O, S, N, & ducatur concurſus N rectarum CON, ANS per rectam NR, & concurſus S rectarum BOS, ANS per rectam QRS; atque punctum O Ellipſin deſcribet ad Diametrum BR, quæ ad ordinatas inclinatur in dato angulo X, & cujus Latus rectum eſt L. Ponamus enim $BA = a$, $AR = CA = AQ = b$, $PB = x$ & $PO = y$, ſitque PO rectæ NR parallela, atque eadem analyſi qua uſi ſumus in Prop. præced. prodibit $y^2 + \frac{x^2 b}{a} + \overline{b+a} \times \frac{-bx}{a} = 0$. Unde colligitur figuram eſſe Ellipſin & ejus Diametrum $a + b = BR$ ſe habere ad Latus rectum, ut a ad b vel BR ad L.



Idem aliter pro Ellipſi. Dividatur BR in D, ut ſit BR : BD : : BR + L : BR − L. In recta BR producta cape BT = BD; dein ſuper Diametris BR & DT, deſcribantur Circuli RVB & DVT, qui ſibi occurrant in V, atque ex V demittatur perpendiculum VA, & rectam BR ſecet in A. Conſtituatur angulus BAC ad punctum A æqualis dato X, & capiatur CA æqualis rectæ AD. Secetur DR in medio G, & ducta GK rectæ CA parallela, centro G

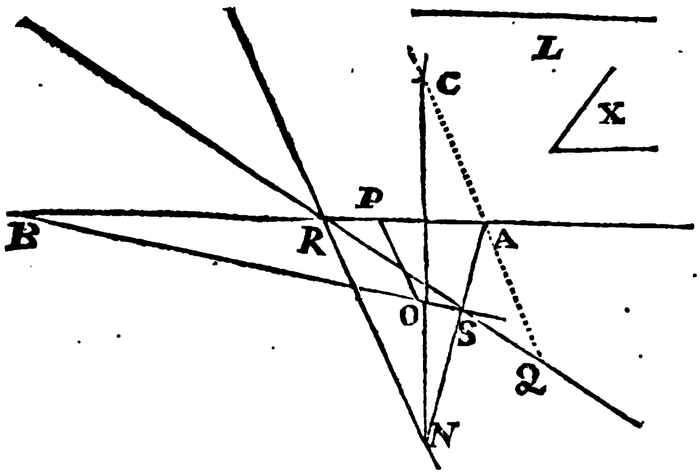
&

& diſtantia GD deſcribatur Circulus rectæ GK occurrens in K; atque per puncta D, K, & R, K agantur rectæ DKS, RKN. Circa polos A, B, C moveantur rectæ ANS, BOS, CON ſibi occurrentes in S, N, O, & ducatur concurſus N rectarum ANS, CON per rectam KR, & concurſus S rectarum BOS, ANS per rectam DK, & punctum O Ellipſin deſcribet ad Diametrum BR, quæ ad ordinatas inclinatur in dato angulo X, Latuſque rectum habet L. Dicatur $AR = b$, $AB = a$, $CA = AD = d$, & jungantur puncta B & V. Quoniam $BR + L : BR - L :: BR : BD = BV$, erit etiam $BR + L : BR - L :: BV : BA :: a + d : a$, atque dividendo & conſequentes duplicando $2L : 2BR - 2L :: d : 2a :: d^2 : 2ad$; & ideo invertendo & componendo $BR : L :: 2ad + d^2 : d^2$. Sed ex natura Circuli $TA \times AD = AVq = BA \times AR$, & igitur $2ad + d^2 = ab$, & proinde $ab : d^2 :: BR : L$. Per punctum O acta ordinata OP rectæ AC parallela, ſitque $BP = x$, & $OP = y$, & orietur $y^2 + \frac{x^2 d^2}{ab} - \frac{d^2 \times \overline{b + a} \times x}{ab} = o$, quæ eſt Æquatio ad Ellipſin cujus Diameter $a + b = BR$, eſt ad Latus rectum ut ab ad d^2. Atqui oſtenſum eſt $2ad + d^2 = ab : d^2 :: BR = a + b : L$, adeoque L erit Latus rectum.

* Pro Hyperbola. Producatur BR & ultra R cape A ut fiat $BA : AR :: BR : L$, atque ad punctum A conſtruatur angulus BAC æqualis dato X, & producta CA ad alteras partes rectæ AB, capiatur $CA = AQ = AR$. Per punctum R ducatur RN rectæ AC parallela, & junctis R & Q, moveantur rectæ ASN, BOS, CON circa polos A, B, C ſibi occurrentes in N, S, O, & ducantur N & S per rectas RN, RQ, atque punctum O Hyperbolam deſcribet ad Diametrum

D BR

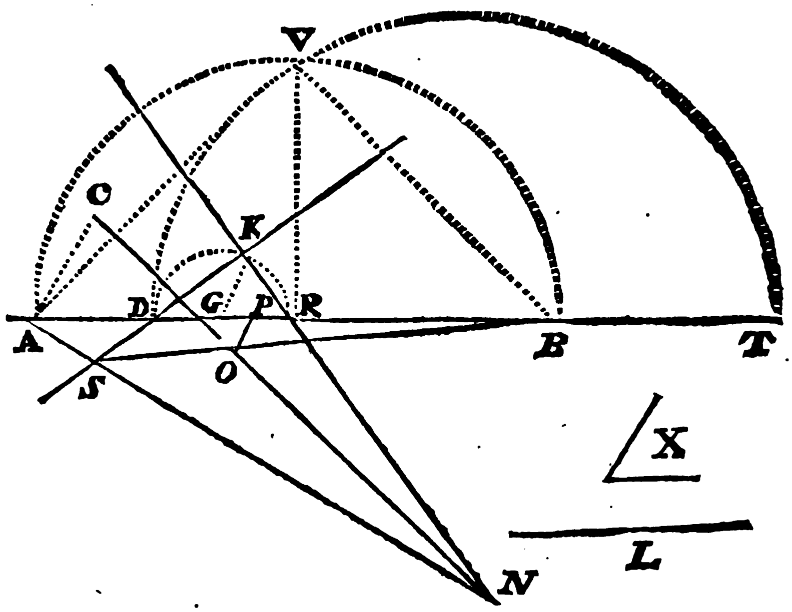
* Vid. fig. ſeq. Pag.



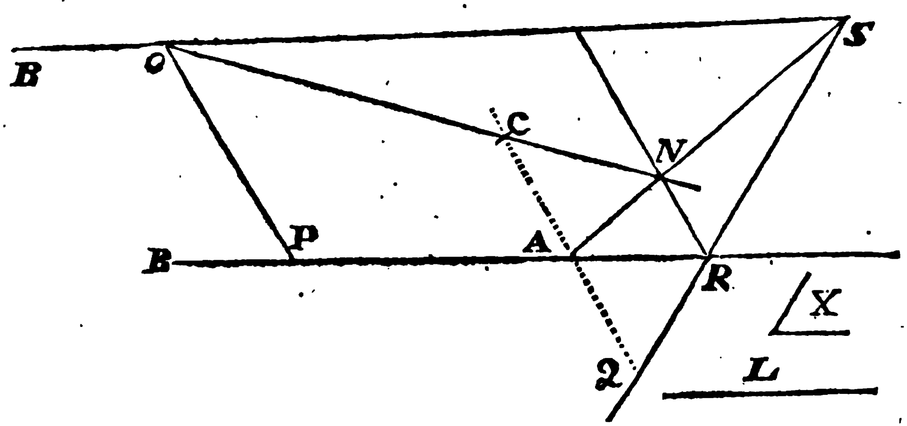
BR & Latus rectum L. Nam iiſdem ſymbolis utendo ut ſupra prodibit. $y^2 - \frac{x^2 b}{a} - \frac{bx}{a} \times \overline{b-a} = 0$, quæ eſt Æquatio ad Hyperbolam cujus Diameter $a-b=$ BR eſt ad ſuum Latus rectum ut a ad b, vel BR ad L.

Idem aliter pro Hyperbola. In recta BR producta cape punctum D ut fiat BR : BD :: BR − L : BR + L, centroque B radio BD deſcribatur Circulus DVT. Ex puncto R erigatur perpendicularis VR, quæ Circulo DVT occurrat in V, & per punctum V ducatur tangens VA ad circulum DVT, quæ rectam BA ſecet in A: dein ad punctum A conſtruatur angulus BAC æqualis dato X, & capiatur CA = AD. Super Diametro DR deſcripto circulo DKR, per centrum G agatur GK rectæ AC parallela circulo DKR occurrens in K, & ducantur rectæ KDS, KRN per puncta D, K, & K, R. Circa polos A, B, C moveantur rectæ ANS, BOS, CON, quæ ſibi occurrant in N, S, O, ducantur N & S per rectas NKR, KDS; & punctum O Hyperbolam deſcribet ad Diametrum BR, quæ ad ordinatas inclinatur in dato angulo

lo X, & habet Latus rectum L. Jungantur puncta B & V, & deſcribatur circulus AVB. Quoniam BR − L : BR + L : : BR : BD = BV erit

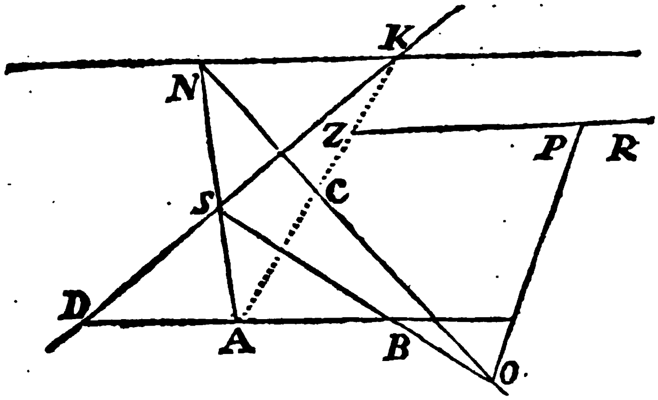


etiam (retentis iiſdem ſymbolis ut ſupra) BR − L : BR + L : : BV : BA : : $a - d : a$; & inde dividendo & conſequentes duplicando 2L : 2BR + 2L : : $d : 2a : : d^2 : 2ad$; atque convertendo & invertendo BR : L : : $2ad - d^2 : d^2$. Porro ex Elem. eſt AB × AR = AVq = AT × AD, ideoque $ab = 2ad - d^2$, & deinde BR : L : : $ab : d^2$. Per punctum O agatur ordinata OP rectæ AC parallela, ſitque PB = x, & PO = y, & prodibit æquatio $y^2 - \frac{d^2x^2}{ab} - \frac{d^2 \times \overline{a-b} \times x}{ab} = o$: unde concluditur figuram eſſe Hyperbolam & ejus Diametrum $a - b$ = BR ſe habere ad ſuum latus rectum ut $ab : d^2$; & ideo L eſt Latus rectum.



Pro Parabola. A vertice R Diametri BR, capiatur AR = L, atque ad punctum A conſtruatur angulus BAC æqualis dato X. In recta CA utrinque producta, ab A ad contrarias partes Diametri BR cape CA & AQ, æquales rectæ AR = L. Per punctum R agatur RN rectæ AC parallela, & jungantur Q & R. Moveantur rectæ OCN, ANS circa polos A, C, ſit vero alter polus B ad diſtantiam infinitam in Diametro RB ad eaſdem partes rectæ RN, atque adeo recta BOS per polum B ducta erit Diametro BR continuo parallela: ducatur concurſus N rectarum ANS, OCN per rectam NR, & concurſus S rectarum ASN, BOS, per rectam QR; & reliquus O Parabolam deſcribet ad Diametrum BR cujus Latus rectum eſt L = AR. Nam ducta ordinata PO rectæ CA parallela, ſit OP = y & PR = x, & proveniet Æquatio $y^2 = L \times x$.

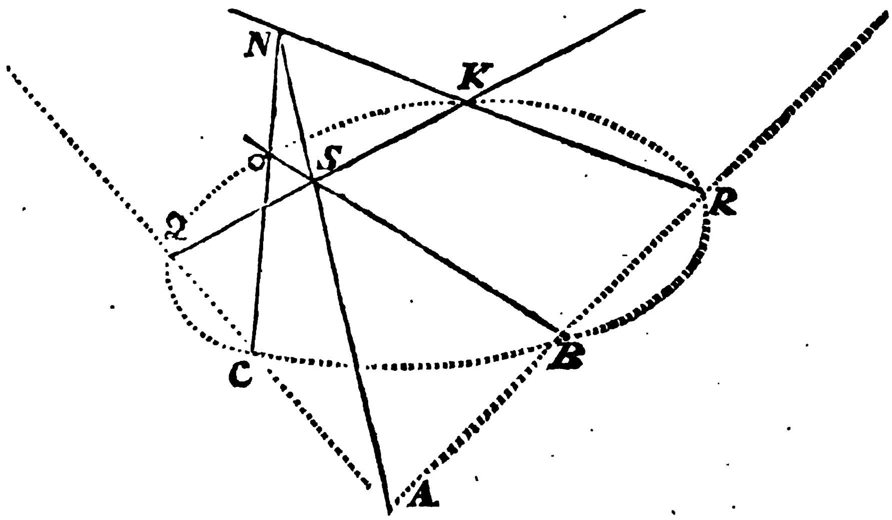
Idem aliter pro Parabola. Sit recta DB æqualis Lateri recto L, ſeceturque in medio A. Et per punctum A ducatur AK rectæ DB æqualis conſtituens angulum BAK æqualem dato X, atque per extremum K agatur KN rectæ DB parallela; & jungantur puncta K, D. Secetur AK in medio C. Dein circa polos A, B, C moveantur rectæ ASN, CNO, BOS, & ducantur concurſus, N & S per rectas NK & DK, & reliquus O Parabolam deſcribet. Dividatur CK in medio Z, & ducta ZR rectis BAD, NK parallela, hæc erit Diameter, quæ



quæ ad ordinatas inclinatur in dato angulo X, & Latus rectum habet L. Nam ducta ordinata OP ad Diametrum ZR rectæ AK parallela; ſit OP $=y$, & ZP $=x$, & orietur Æquatio $y^2 = \frac{L^2}{16} + L \times x$.

PROP. III.

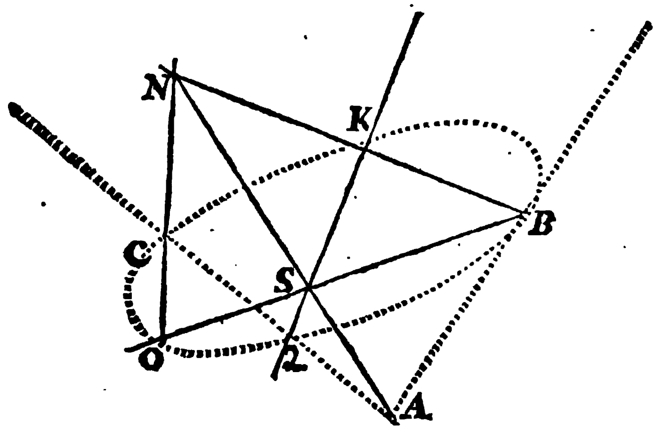
PER quinque puncta data Sectionem Conicam deſcribere.



Sunto data quinque puncta B, C, K, Q, R. Ducantur duæ rectæ KQ & RK per tria data K, Q, R, ſibi occurrentes in dato K; &

& per puncta R, B, & Q, C, agantur rectæ RB & QC quæ conveniant in A. Circa polos A, B, C moveantur rectæ ASN, BSO, CON, quæ ſibi occurrant in S, N, O; ducatur concurſus S rectarum ASN, BSO per rectam QK, non quidem per rectam KR, ex cujus puncto dato R ducta fuit ABR; rectarum vero ASN, CNO concurſus N per rectam NKR non autem per QK ex cujus puncto dato Q acta erat QCA; atque punctum O rectarum CON, BSO concurſus Sectionem Conicam deſcribet, per quinque puncta data B, C, K, Q, R tranſeuntem. Hæc Conſtructio demonſtratur ex Corollarii 1 & 3, Prop. 1ᵐᵃ.

COROL. I.

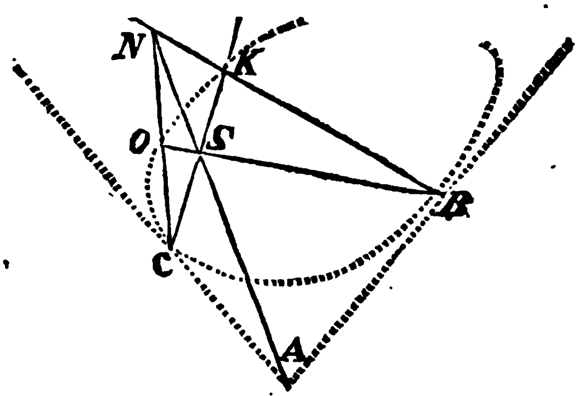


Hinc deſcribi poteſt Sectio Conica, per quatuor puncta data, quorum unum ſit in data tangente. Eſto enim BA tangens data, punctumque B in ea datum, atque reliqua tria C, Q, K. Ex puncto K ducantur per B & Q, rectæ BK, QK, & junctis Q & C, producatur recta QCA quæ tangenti occurrat in A; dein circa polos A, B, C moveantur rectæ ASN, BSO, CON, ſibi occurrentes in N, S, O; ducatur N concurſus rectarum CON & ASN per rectam BK, & concurſus S rectarum

ASN

ASN, BSO, per rectam QK; & punctum O Sectionem Conicam deſcribet quæ per data K, Q, C tranſibit, rectamque A B continget in dato B. Per Corol. 3. Prop. 1.

COROL. II.



Quinetiam deſcribi poteſt Sectio Conica per tria puncta data B, K, C, quorum duo ſunt in datis tangentibus AC & AB. Nam conveniant tangentes A B & C A in A, & per puncta B, C, in iis data ducantur ad reliquum K rectæ BK, CK; deinde circa polos A, B, C moveantur rectæ ASN, BSO, CON, ſibi occurrentes in N, S, O; ducatur concurſus N rectarum ASN, CON per rectam BK, & concurſus S rectaram ANS, BSO, per rectam CK; atque reliquus O Sectionem Conicam deſcribet quæ tranſibit per datum K & rectas BA, AC in datis B, C continget. Corol. 3. Prop. 1.

SECTIO

SECTIO II.

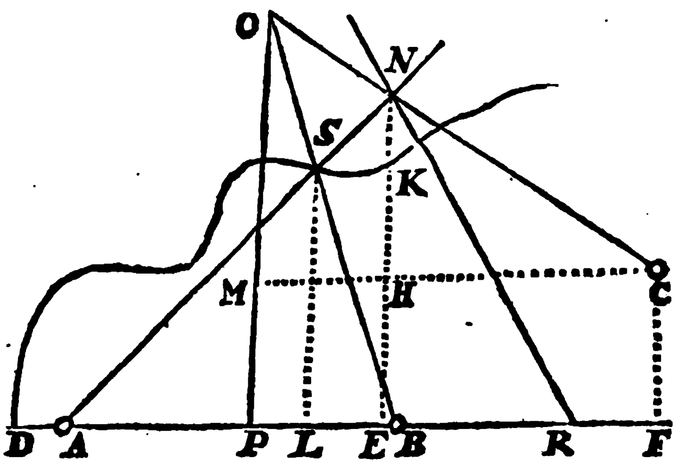
De deſcriptione Linearum cujuſcunque Ordinis ope Linearum Ordinis inferioris.

PROP. IV.

MOveantur tres rectæ ASN, BSO, CNO in plano quovis, circa tria puncta data A, B, C, ſeſe interſecantes in totidem punctis N, S, O; ducatur unum S per datam Lineam DSK ordinis n, *& alterum N per rectam poſitione datam KN; dico reliquum O Lineam ordinis* 2n *deſcribere.*

Per puncta data A & B agatur recta DABR, quæ Lineæ DSK ordininis n occurrat in D, & poſitione datæ RKN in R, & ſit Curvæ deſcribendæ ordinata PO in dato quovis angulo ad rectam DABR inclinata eamque ſecet in P; atque ex punctis C, N, S ducantur rectæ CF, NE, SL, ordinatæ OP parallelæ, quæ rectæ DABR occurrant in F, E, L. Denique ex dato C age rectam CMH rectæ DABR parallelam, & rectas PO & NE ſecantem in M & H. Igitur datæ ſunt CF, AB, AR, AF, AD. Ponatur $AB = a$, $AR = b$, $CF = c$, $AF = f$, abſciſſa Curvæ deſcribendæ $AP = x$, ordinata $PO = y$, abſciſſa Curvæ datæ $AL = z$, ordinata $LS = s$, recta $AE = u$; & quia triangulum REN eſt ſpecie datum ſit $RE : EN :: b : g$. Et inde $EN = \frac{gb - ug}{b}$. Generalis Æquatio Curvæ datæ DSK erit hujus formæ $S^n + \overline{Az + B} \times S^{n-1} + \overline{CZ^2 + DZ + E} \times S^{n-2} + \overline{Fz^3 + Gz^2 + Hz + I} \times S^{n-3} + \overline{Kz^4 + Lz^3 + Mz^2 + Nz + V} \times S^{n-4} + \overline{Pz^5 + Qz^4 + Rz^3 + Tz^2 + Wz + X} \times S^{n-5} + \&c. = 0$. Et ideo ſi

ſi exterminentur Z & S, ſubſtituendo earum valores quantitatibus x & y involutos, producetur Æquatio Curvæ deſcribendæ: eliminari autem poſſunt Z & S analyſi ſequenti. Triangula CMO & CHN ſunt æquiangula & ob CM : MO :: CH : HN erit $f-x : y-c :: f-u : \frac{gb-ug}{b}-c$ adeoque $\overline{y-c} \times \overline{f-u} = \overline{f-x} \times \overline{\frac{gb-ug-bc}{b}}$, & inde $u = \frac{xgb+byf-bgf-xbc}{xg+by-fg-bc}$. Et quia Triangula ASL & ANE ſunt



etiam ſimilia & AL : LS :: AE : EN erit $z : s :: u : \frac{gb-ug}{b}$, & proinde $\frac{zgb}{zg+sb} = u$. Æquentur duo valores reperti rectæ AE $= u$, & ducendo unius numeratorem in denominatorem alterius prodibit $\overline{xgb+byf-bgf-xbc} \times \overline{zg+sb} = \overline{xg+by-fg-bc} \times zgb$; & ideo $S = \frac{\overline{yb-yf-bc+xc} \times zg}{xgb+byf-bgf-xbc}$. Cæterum Triangula BSL & BPO ſunt æquiangula, & BL : LS :: BP : PO, atque ita erit

$a-z : s : a-x : y$, & $s=\frac{ay-zy}{a-x}$. Igitur ſi æquentur etiam duo valores rectæ $s=LS$, erit $\overline{yb-yf-bc+xc}\times zg\times\overline{a-x}=\overline{a-z}\times y\times\overline{xgb+byf-bgf-xbc}$: unde

$$z=\frac{ayb\times\overline{xg+yf-gf-xc}}{\overline{a-x}\times g\times\overline{yb-yf-bc+xc}+by\times\overline{xg+fy-fg-xc}},$$

& deinde

$$s=\frac{ayg\times\overline{by-yf-cb+xc}}{\overline{a-x}\times g\times\overline{yb-yf-bc+xc}+by\times\overline{xg+fy-fg-xc}}.$$

In æquatione generali ſubſtituantur valores jam inventi rectarum Z & S, & prodibit Æquatio

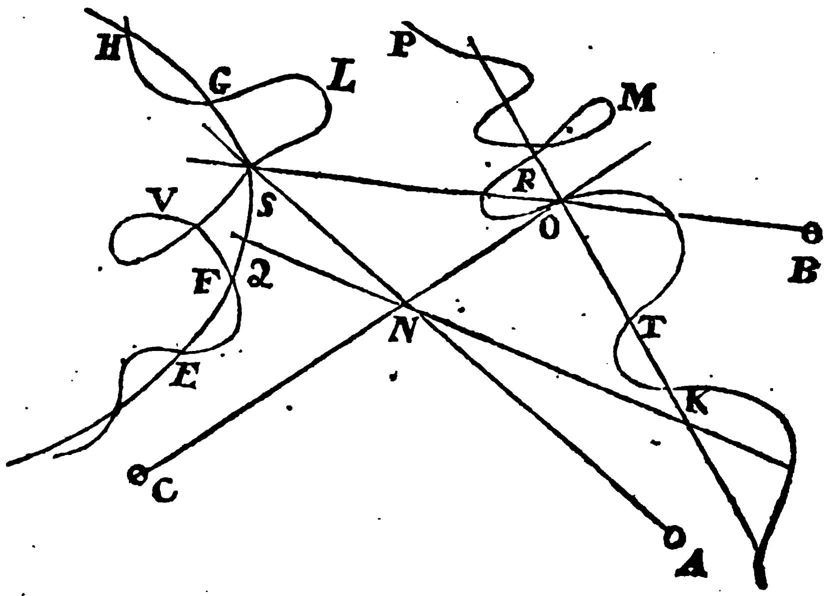
$$a^{n}y^{n}b^{n}\times\overline{by-yf-cb+xc}\,|^{n}+A\,a^{n}y^{n}g^{n-1}\times\overline{by-yf-cb+xc}\,|^{n-1}\times b\times\overline{xg+yf-gf-xc}+B\,a^{n-1}y^{n-1}g^{n-1}$$
$$\times\overline{by-yf-cb+xc}\,|^{n-1}\times\overline{\overline{a-x}\times g\times\overline{yb-yf-bc+xc}+by\times\overline{xg+fy-fg-xc}}$$
$$+C\,a^{n}y^{n}g^{n-2}\times\overline{by-yf-cb+xc}\,|^{n-2}\times b^{2}\times\overline{xg+yf-gf-xc}\,|^{2}$$
$$+D\,a^{n-1}y^{n-1}g^{n-2}\times\overline{by-yf-cb+xc}\,|^{n-2}\times b\times\overline{xg+yf-gf-xc}$$
$$\times\overline{\overline{a-x}\times\overline{yb-yf-bc+xc}+by\times\overline{xg+fy-fg-xc}}+E\,a^{n-2}y^{n-2}g^{n-2}$$
$$\times\overline{by-yf-cb+xc}\,|^{n-2}\times\overline{\overline{a-x}\times\overline{yb-yf-bc+xc}+by\times\overline{xg+fy-fg-xc}}\,|^{2}$$
$$+\,\&c.=0.$$

Ergo cum Æquatio ad totidem dimenſiones aſcendat quot ſunt unitates in numero $2n$, patet Lineam motu puncti O deſcriptam eſſe ordinis $2n$.

Idem Aliter.

Sit EFVLGH Linea data ordinis n, & ducatur recta PRT in plano Lineæ motu puncti O deſcriptæ, & inveſtigandum ſit in quot punctis Curvæ hæc recta occurrat. Si punctum O moveretur per reccam RTK, & interea alterum N per rectam QNK poſitione datam; reliquum S Sectionem Conicam HGSQFE deſcriberet, quæ datæ Lineæ EFVLGH ordi-

ordinis n occurrat in punctis H, G, F, E, *&c.* & *e* contra ſi illud S duceretur per jam deſcriptam Sectionem Conicam HGSQFE & alterum N per rectam KN; reliquum O deſcriberet rectam RTK. Moveatur jam punctum S per Lineam EFVLGH datam ordinis n, alterum vero N



per rectam QNK, & quoties punctum S occurret interſectionibus, H, G, F, E, *&c.* Sectionis Conicæ & Lineæ ordinis n; reliquum O erit in recta RTK: ſi vero S ſit in aliis Curvæ EFVLGH punctis, non erit punctum O in recta RTK, ſed Curvam PMRT deſcribet. Cum igitur Sectio Conica occurrere poteſt Lineæ ordinis n in punctis $2n$, manifeſtum eſt Lineam PMROT motu puncti O deſcriptam rectæ RTK occurrere in punctis $2n$. Adeoque quia in tot punctis Curva quævis rectam lineam ſecare poteſt quot ſunt unitates in numero indicis ſuæ, patet Lineam motu puncti O deſcriptam eſſe ordinis $2n$.

COROL. I.

* Curva methodo hujus Prop. deſcripta toties tranſire poteſt per datum punctum R interſectionis rectarum NKR & ABR quot ſunt unitates in *n*. Nam Linea data DSK rectæ ABR occurrere poteſt in *n* punctis & proinde punctum S interſectio rectarum BSO & ASN movendo attingat rectam ABR in totidem punctis; ſed quando punctum illud S ſit in recta ABR, Linea motu puncti O deſcripta rectam ABR decuſſabit in puncto R.

COROL. II.

Curva etiam toties tranſire poteſt per polum C quot ſunt unitates in *n*. Nam ſi ducatur recta per polos B & C, Curva DSK hanc rectam ſecare poteſt in punctis quorum numerus eſt *n*, & cum concurſus S rectarum ASN, BSO percurrendo Curvam DSK perveniat ad interſectiones rectæ & Curvæ punctum deſcribens O erit in polo C.

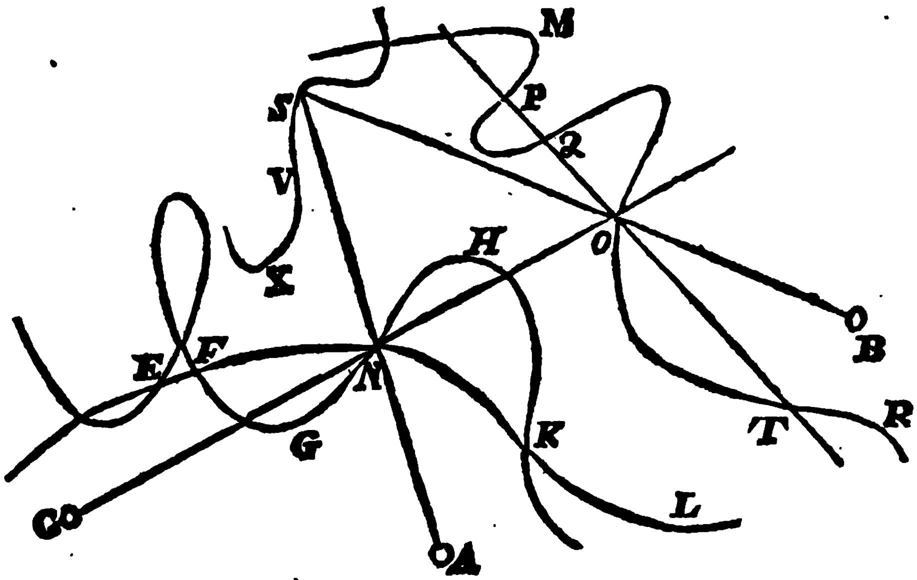
PROP. V.

Iiſdem poſitis: ducantur duo interſectionum puncta S & N per Lineas datas XVS & ENKL, ordinum n *&* m; *dico reliquum O Lineam MPQR ordinis* 2 n m *deſcribere.*

In plano Lineæ motu puncti O deſcriptæ agatur recta PQT, & inveſtigandum ſit quoties curva huic rectæ occurrere poſſit. Ponamus punctum O duci per rectam PQT, & interea alterum S per Lineam datam XVS ordinis *n*; & reliquum N deſcriberet per Prop. præced. Lineam EFGH ordinis 2 *n*, quæ Lineæ ENKL datæ ordinis *m* occurrat in punctis E, F, N, K, &c. & viciſſim ſi duceretur punctum N per Lineam jam deſcriptam EFGH ordinis 2 *n*, & alterum S per

* Vid. fig. Pag. 25.

per datam Lineam XVS ordinis *n*; reliquum O percurreret rectam PQT. Moveatur nunc punctum N per datam Lineam ENKL ordinis *m*, atque alterum S per Lineam datam XVS ordinis *n*, & quo-



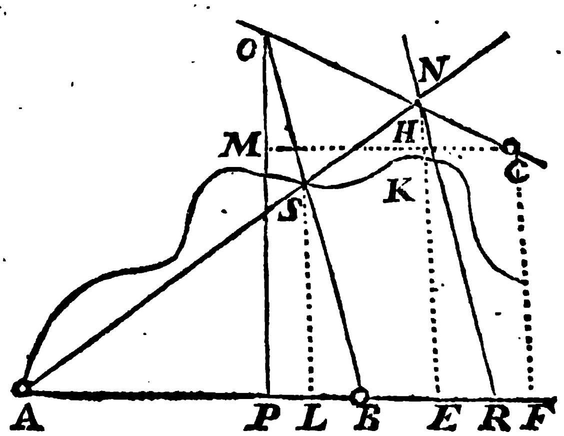
ties punctum N occurrat interſectionibus E, F, N, K, &c. Linearum EFGH & ENKL, ordinum 2*n* & *m*; punctum O erit in recta PQT. Si autem N ſit in aliis Curvæ EFKL punctis, non erit punctum O in recta PQT. Sed Linea ENKL ordinis *m* occurrere poteſt Lineæ ordinis 2*n* in punctis 2*nm*; & ideo conſtat Lineam MQOTR motu puncti O deſcriptam in totidem punctis rectam PQT ſecare, & eam eſſe ordinis 2*nm*.

PROP.

PROP. VI.

CIRCA tria puncta data A, B, C, in plano quovis tanquam polos moveantur tres rectæ ASN, BSO, CNO ſibi occurrentes in tribus punctis S, N, O; ducatur unum N per rectam NR poſitione datam & alterum S per Lineam datam ASK, ordinis n *per polum A tranſeuntem; dico reliquum O deſcripturum Lineam ordinis* 2n − 1.

Per polos A & B quorum unus A eſt in Curva ASK, agatur recta AB rectæ KR poſitione datæ occurrens in R, & a puncto deſcribente O ſit ordinata OP, rectam ABR ſecans in P, & cum ea datum angulum conſtituens. A punctis C, S, N, ducantur rectæ CF, SL, NE quæ ordinatæ OP ſint parallelæ ſecentque rectam ABR in F, L, E; dein per polum C agatur CMH rectæ ABR parallela, rectiſque OP, NE



occurrat in M & H. Ideoque dantur rectæ CF, AB, AR, AF. Dicatur $AB = a$, $AR = b$, $CF = c$, $AF = f$. Abſciſſa Curvæ deſcribendæ $AP = x$, ordinata $OP = y$, abſciſſa Curvæ datæ $AL = z$, ordinata $LS = s$, & propter triangulum REN ſpecie datum fiat $RE : EN :: b : g$. Quoniam polus A a quo incipit abſciſſa $AL = z$ ſit

ſit in ipſa Curva A S K, conſtat generalem Æquationem non habere terminum abſolute cognitum, & proinde Æquatio ordinis n Curvæ A S K erit hujus formæ

$$\begin{array}{l} + A z + B z^2 + C z^3 \ldots\ldots\ldots\ldots + D z^n \\ + \overline{E + F z + G z^2 \ldots\ldots\ldots\ldots + H z^{n-1}} \times S \\ + \overline{K + L z + M z^2 \ldots\ldots\ldots\ldots + N z^{n-2}} \times S^2 \\ + \overline{P + Q z + R z^2 \ldots\ldots\ldots\ldots + T z^{n-3}} \times S^3 + \&c. = 0. \end{array}$$

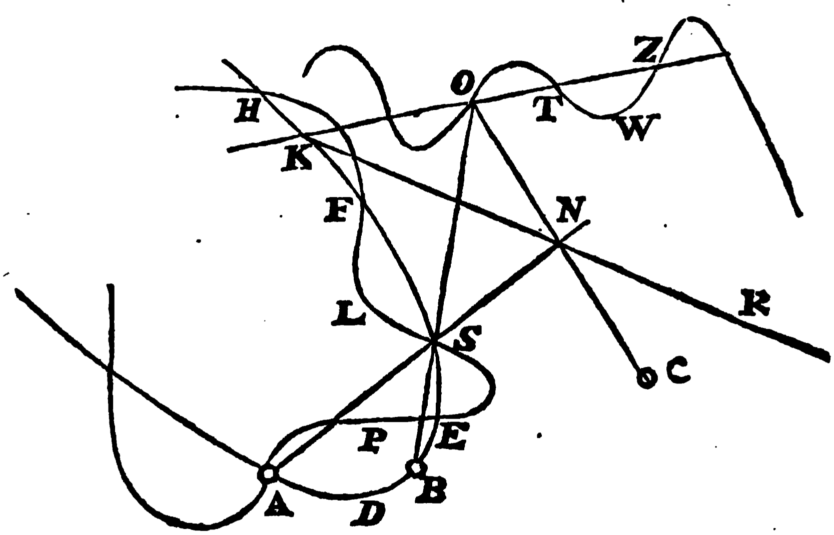
In qua nullus eſt Terminus qui non ducatur in Z vel S. Coefficientes A, B, D, E, &c. ſunt quantitates cognitæ. Sed eadem analyſi qua uſi ſumus in Prop. 4. invenire licet $Z = \dfrac{ayb \times \overline{xg + yf - gf - xc}}{\overline{a - x} \times g \times \overline{yb - yf - bc + xc} + by \times \overline{xg + yf - fg - xc}}$ & $S = \dfrac{ayg \times \overline{by - yf - cb + xc}}{\overline{a - x} \times g \times \overline{yb - yf - bc + xc} + by \times \overline{xg + fy - fg - xc}}$; & hi valores rectarum z & s eundem Denominatorem habent, ac uterque Numerator ductus eſt in y. Subſtituantur valores reperti in Æquatione generali Curvæ A S K, & prodibit Æquatio in qua x & y ad totidem dimenſiones aſcendent quot ſunt unitates in $2n$, & omnis terminus ductus erit in y. Dividantur termini per y & emerget Æquatio ordinis $2n - 1$. Adeoque Linea motu puncti O deſcripta eſt ordinis $2n - 1$.

Idem Aliter.

* Eſto enim A P E L H data Linea ordinis n, & in plano Lineæ OTWZ motu puncti O deſcriptæ ducatur recta OTZ, & inveſtigandum ſit in quot punctis Curva OTWZ, hanc rectam ſecet. Si duceretur punctum O per rectam OTZ, & interea alterum N per rectam KNR poſitione datam; reliquum S deſcriberet per Prop. 1. Sectionem Conicam ADBSK, per data puncta A & B tranſeuntem, quæ Lineæ ordinis n ocurrat in $2n$ punctis A, E, S, F, H, &c. & vice verſa ſi punctum S moveretur per Sectio-

* Vid. fig. ſeq. Pag.

Sectionem Conicam ADBSK jam deſcriptam atque alterum N per rectam KNR, reliquum O percurreret rectam OTZ. Moveatur nunc punctum S per Lineam datam APESLH ordinis n, (quæ tranſeat per polum A) & alterum N per rectam KNR, & quoties punctum S occurrat interſectionibus E, S, F, H, &c. Curvæ & Sectionis Conicæ, punctum O erit in recta OTZ, at non ſæpius; excepto tantum caſu



cum punctum S tranſeat per polum A, ibi enim recta ASN tanget Curvam datam APELH ordinis n, & punctum O non erit in recta OTZ, niſi altera interſectio E coincidat cum A, atque Sectio Conica & Linea ordinis n ſeſe tangant in A, adeoque cum punctum S occurrat interſectionibus E, S, F, H, &c. quarum numerus eſt $2n-1$, punctum O erit in recta OTZ; & inde Curva OTW rectam ſecabit in $2n-1$ punctis, at non in pluribus. Ergo Linea motu puncti O deſcripta eſt ordinis $2n-1$.

COROL.

COROL. I.

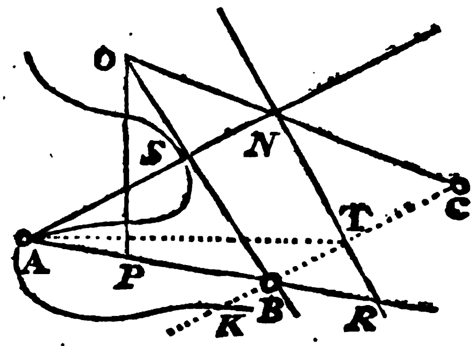
Si per polos A & B ducatur recta B A ut rectæ N R productæ occurrat in R; Curva deſcripta tot habeat arcus in puncto R quot ſunt unitates in numero $n-1$. Nam Linea data H L S P A ordinis n rectam A B ſecare poteſt in n punctis, & ideo punctum S interſectio rectarum ASN, BSO, movendo per Lineam H L S P A rectam B A attinget in totidem punctis; dum vero illud S ſit in recta B A coincident rectæ B S O, AS N cum recta B A; excepto tantum caſu quando S percurrat punctum A, ibi enim rectæ ASN, BSO non coincident ſed recta ANS erit tangens Lineæ datæ H L S P A, & proinde punctum O Curvæ deſcriptæ interſectionem R rectarum B A, N R non attinget. Adeoque toties Curva deſcripta tranſibit per punctum R quot ſunt unitates in numero $n-1$. Et ſimiliter ſi concurſus N rectarum A S N, C N O, moveatur per Lineam datam ordinis n, concurſus vero S rectarum ASN, BSO, per rectam poſitione datam, & agatur recta per polos A, C quæ poſitione datæ occurrat in Q; Curva deſcripta habere poteſt $n-1$ puncta in Q.

COROL. II.

Curva deſcripta toties tranſire poteſt per polum C quot ſunt unitates in indice n. Agatur enim recta per polos B, C, & Linea data HLSPA ordinis n huic rectæ occurrere poteſt in n punctis; quoties autem concurſus S movendo per Lineam H L S P A ordinis n perveniat ad interſectiones rectæ BC, & Lineæ H L S P A, punctum deſcribens O locabitur in polo C. Et pariter ſi ducatur concurſus N rectarum A S N, C N O per Lineam datam ordinis n, & concurſus S per rectam poſitione datam; Curva deſcripta habere poteſt n puncta in polo B.

COROL. III.

Quinetiam Curva deſcripta habere poteſt $n-1$ puncta in polo B. Agatur recta per polos B, D, quæ poſitione datæ NR occurrat in T & junguntur puncta A, T. Quoniam Linea data KAS ordinis n rectæ AT occurrere poteſt in punctis n; concurſus S rectarum ASN, BSO movendo per Curvam illam KAS tranſcurret rectam AT in totidem punctis, & ideo concurſus alter N rectarum ASN, CNO toties coincidet



cum puncto T, excepto tantum caſu quando concurſus S tranſit per polum A, ibi enim recta ASN erit tangens Curvæ datæ KAS, atque adeo puncta N & T toties tantum coincident quot ſunt unitates in numero $n-1$. Coeuntibus vero N, T, coincident rectæ CNO & BC, punctumque O locabitur in polo B. Adeoque Curva motu puncti O deſcripta habere poteſt $n-1$ puncta in polo B. Et ſimili argumento ſi ducatur concurſus N per Lineam datam ordinis n, & concurſus S per rectam poſitione datam; Curva deſcripta tantum habere poteſt $n-1$ puncta in polo C.

COROL. IV.

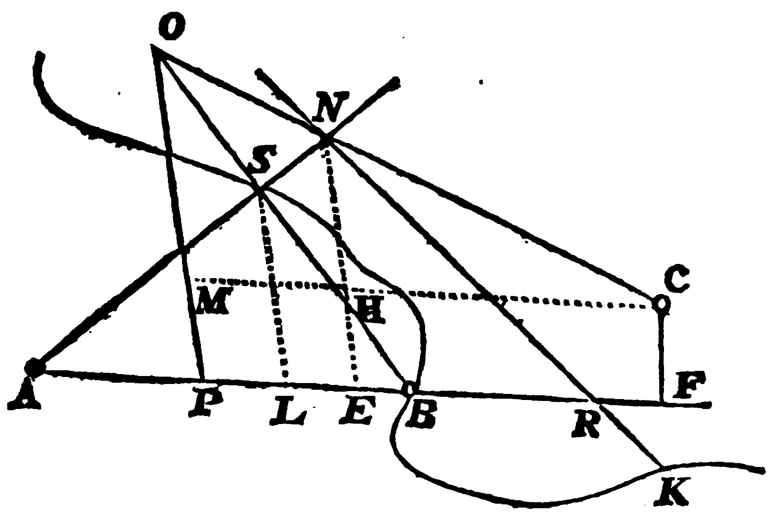
Si Curva data KASL ſit Sectio Conica, evadet Index $n=2$, atque Curva deſcripta ſemel tranſibit per puncta B, R, & dupliciter per C.

PROP.

PROP. VII.

Iiſdem poſitis moveatur concurſus N rectarum ASN, CNO per rectam KR poſitione datam, & concurſus S rectarum ASN, BSO per Lineam datam KBS ordinis n tranſeuntem per polum B, ex quo ducitur recta BSO; vel moveatur concurſus S rectarum ASN, BSO per rectam poſitione datam, & concurſus N rectarum ASN, CNO per Lineam datam ordinis n, *tranſeuntem per polum C, ex quo ducta eſt recta CNO: dico in utroque caſu reliquum concurſum O deſcripturum Lineam ordinis* 2n − 1.

Imprimis moveatur concurſus S rectarum ASN, BSO per Lineam KBS ordinis *n* per polum B tranſeuntem. Agatur recta per polos A, B, quæ poſitione datæ NR occurrat in R, & ex puncto deſcribente O acta ordinata OP ad rectam AB in dato angulo inclinata, per puncta C, S, N, ducantur rectæ CF, SL, NE ordinatæ OP parallelæ, & quæ rectæ



AB occurrant in F, L, E; atque per polum C agatur CMH rectæ AB parallela rectaſque OP, NE, ſecet in M, H. Dicatur AB = *a*, AK = *b*,

 AF = *f*

$AF = f$, $CF = c$, $AE = u$, Curvæ datæ KBS ordinata $SL = s$, abſciſſa $BL = z$, Curvæ deſcribendæ ordinata $OP = y$, abſciſſa $AP = x$, & quia dantur anguli trianguli REN, ſit $RE : EN :: b : g$, & inde $NE = \frac{gb - gu}{b}$. Quoniam igitur triangula ALS, AEN ſunt æquiangula & $AL : LS :: AE : EN$ erit $a - z : s :: u : \frac{bg - ug}{b}$, atque $u = \frac{abg - zbg}{bs + ag - zg}$. Et ob triangula CHN, CMO, ſimilia, & $CH : HN :: CM : MO$, fiet $f - u : \frac{bg - ug - bc}{b} :: f - x : y - c$, & proinde $u = \frac{yfb - fbg + bgx - bcx}{by - bc - fg + xg}$; atque adeo $\frac{abg - zbg}{bs + ag - zg} = \frac{yfb - fbg + bgx - bcx}{by - bc - fg + xg}$: unde $z = \frac{\overline{yf - fg + gx - cx} \times bs + \overline{yf - cx - by + bc} \times ag}{\overline{yf - cx - by + bc} \times g}$. Præterea triangula BSL, BPO ſunt ſimilia, ac $BL : LS :: BP : PO$, & deinde $z : s :: a - x : y$; adeoque $z = \frac{s \times \overline{a - x}}{y}$. Æquentur valores abſciſſæ $z = BL$, & propterea emerget Curvæ datæ KBS ordinata $S = \frac{yag \times \overline{yf - xc - by + bc}}{\overline{a - x} \times g \times \overline{yf - cx - by + bc} - yb \times \overline{yf - fg + gx - xc}}$, & inſuper abſciſſa $z = \frac{\overline{a - x} \times ag \times \overline{yf - cx - by + bc}}{\overline{a - x} \times g \times \overline{yf - cx - by + bc} - yb \times \overline{yf - fg + gx - cx}}$. Subſtituantur valores reperti ordinatæ S, & abſciſſæ Z Curvæ datæ KBS in in æquatione generali $\overline{Az + Bz^2 + Cz^3 \ldots\ldots Dz^n} + S \times \overline{E + Fz + Gz^2 \ldots\ldots Hz^{n-1}} + S^2 \times \overline{K + Lz + Mz^2 \ldots\ldots Nz^{n-2}} + S^3 \times \overline{P + Qz + Rz^2 \ldots\ldots\ldots Tz^{n-3}} + \&c.$ Ubi deeſſe debet quantitas abſolute cognita, & producetur Æquatio ordinis $2n$ cujus

termini omnes ducentur in $yf - cx - by + bc$. Dividantur termini per $yf - xc - by + bc$, & prodibit Æquatio ordinis $2n - 1$. Ideoque Linea motu puncti O deſcripta erit ordinis $2n - 1$. Moveatur jam concurſus N rectarum ASN, CNO per Lineam datam ordinis n tranſeuntem per polum C & concurſus S per rectam poſitione datam; atque eadem analyſi mutatis mutandis patebit Lineam motu puncti O deſcriptam eſſe ordinis $2n - 1$. Demonſtratur etiam ad modum poſteriorem Prop. præced.

COROL. I.

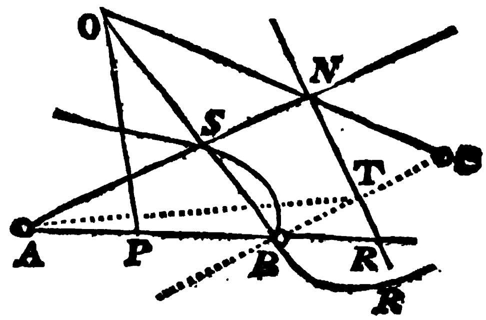
Curva deſcripta tot habere poteſt arcus in R quot ſunt unitates in numero $n - 1$. Nam Linea KBS occurrere poteſt rectæ AB in punctis n; & cum concurſus S movendo per Lineam KBS decuſſet rectam AB, punctum deſcribens erit in R, niſi caſum excipias quando S tranſeat per polum B; adeoque Curva habebit $n - 1$ puncta in R. Et ſimili ratiocinio ſi concurſus N rectarum ASN, CNO percurrat Lineam datam ordinis n per polum C tranſeuntem, alter vero S rectam poſitione datam, atque agatur recta per polos A, C, quæ poſitione datæ occurrat in Q; Curva deſcripta toties tranſibit per punctum Q quot ſunt unitates in $n - 1$.

COROL. II.

Curvæ arcus $n - 1$ concurrere poſſunt in polo C. Jungantur enim poli B & C, & Linea data ordinis n hanc rectam ſecet in n punctis; & ideo quoties concurſus S movendo per Lineam datam KBS attingat rectam BC punctum O locabitur in polo C excepto tantum caſu quando S perveniat ad polum B; nam ibi recta BSO erit tangens Lineæ datæ KBS & proinde punctum O non erit in recta BC; adeoque Curva deſcripta habebit $n - 1$ arcus in polo C. Et ſimiliter ſi concurſus N rectarum ASN, CNO agatur per Lineam datam ordinis n, tranſeuntem per polum C, concurſus vero S per rectam poſitione datam Curva deſcripta habebit $n - 1$ puncta in polo B.

COROL. III.

Curva deſcripta etiam habere poteſt *n* puncta in polo B. Ducatur recta per polos B, C, quæ poſitione datam N R ſecet in T, & jungantur puncta A, T. Quoniam Linea data K B S ordinis *n* rectam T A ſecare poteſt in punctis *n*, concurſus S movendo per Lineam K BS decuſſabit rectam AT in totidem punctis atque ita concurſus N toties coincidet cum puncto T. Quando autem concurſus N attingat punctum



T coincident rectæ CNO & BC, ac punctum deſcribens O tranſibit per polum B. Et igitur cum toties perveniat concurſus N ad punctum T quot ſunt unitates in indice *n*, liquet punctum deſcribens O tot vicibus per polum B tranſire. Adeoque Curva motu pnncti O deſcripta habere poteſt *n* puncta in polo B. Et pariter ſi ducatur concurſus N per datam Lineam ordinis *n* tranſeuntem per polum C & concurſus S per rectam poſitione datam; Curva deſcripta habere poteſt *n* puncta in polo C.

COROL. IV.

Si Linea data KBS ſit Sectio Conica, evadet index $n = 2$, & ideo Curva deſcripta ſemel tranſibit per puncta C & R & dupliciter per C.

Scholium.

Scholium.

In prop. 4. methodum demonſtravimus qua deſcribi poſſunt Lineæ quarum index eſt numerus par, atque ex duabus noviſſimis Prop. conſtat methodus Lineas deſcribendi quarum index eſt impar; & ideo theoriam perficimus Lineas deſcribendi cujuſcunque ordinis ope Linearum ordinis inferioris. Speciatim vero liquet quomodo deſcribi poſſunt facillime Lineæ tertii & quarti ordinis ope Sectionis Conicæ. Opus haud difficile aggrederer, ſi in hoc loco Linearum tertii ordinis ſpecies detegerem pro vario ſitu & ſpecie Sectionis Conicæ; ſed vix operæ pretium erit, cum vereor ne dormitet lector perſcrutando prolixum calculum nullius fere uſus. Qui plura vult ipſe adeat calculum, & forſan ſuo majus delectabitur, quam ſi meum legeret.

PROP. VIII.

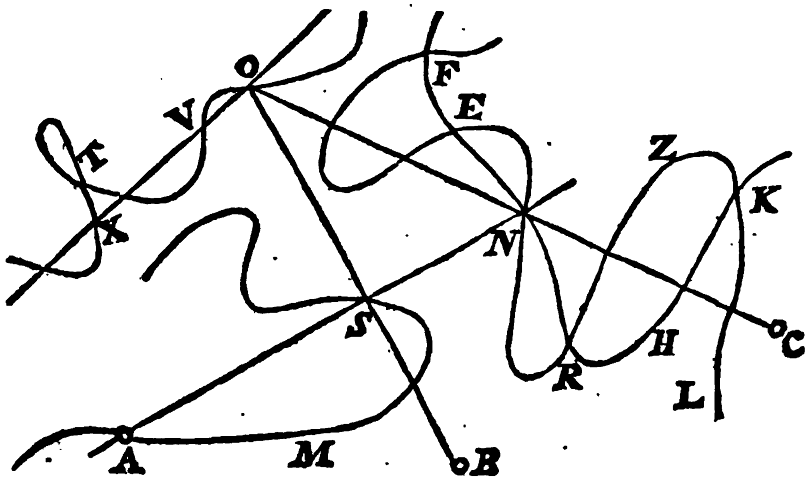
ISdem poſitis ut in Prop. 6. ducantur concurſus S & N per Lineas datas SMA & ERZL ordinis n *&* m, *quarum una SMA tranſeat per polum A, atque concurſus O deſcribet Lineam ordinis* 2nm − m.

* In plano Lineæ XTVO motu puncti O deſcriptæ agatur recta XVO, & quæratur in quot punctis Curvæ hæc recta occurrat. Ponamus punctum O moveri per rectam XV, & interea alterum S per Lineam datam SAM ordinis *n* quæ tranſeat per polum A; & per Prop. præced. reliquum N deſcribet Lineam FNHK ordinis 2*n* − 1, quæ datæ Lineæ ERZL ordinis *m* occurrat in punctis F, E, N, R, K, &c. quorum numerus eſt 2*nm* − *m*: & viciſſim ſi moveatur punctum N per Lineam FNHK jam deſcriptam ordinis 2*n* − 1, & alterum S per datam Lineam SAM ordinis *n*; reliquum O percurret rectam XVO.

Movea-

* Vid. fig. ſeq. pag.

Moveatur nunc punctum N per Lineam datam ERZL ordinis *m*, & alterum S per datam AMS ordinis *n*; & quoties punctum N tranſeat per interſectiones F, E, N, R, K, *&c.* Linearum ERZL &



ENHK ordinum *m* & $2n-1$, punctum O erit in recta XVO: & cum N ſit in aliis punctis Lineæ datæ ERZL ordinis *m* punctum O non tanget rectam XVO. Sed interſectionum numerus quibus ſeſe interſecent Curvæ ERZL, FNHK, eſt $2nm-m$; adeoque Linea XTVO motu puncti O deſcripta in totidem punctis rectam XVO ſecabit: & inde patet Lineam eſſe ordinis $2nm-m$.

PROP. IX.

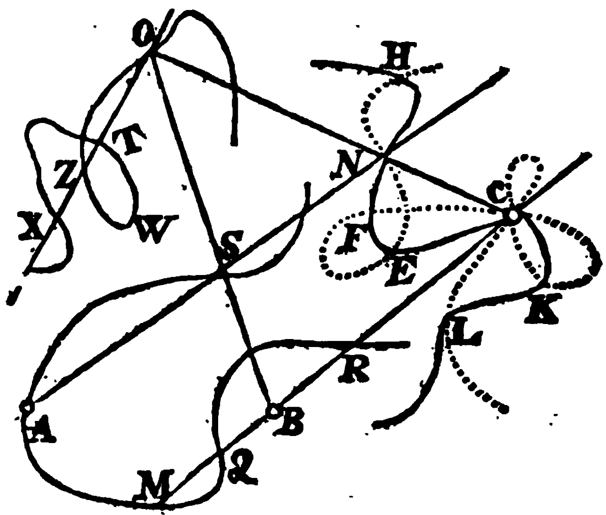
Poſitis quæ in Prop. 7. *Moveatur concurſus S rectarum ASN, BSO, per Lineam ordinis* n *tranſeuntem per polum B, & concurſus N rectarum ASN, CNO per Lineam ordinis* m; *vel moveatur concurſus N per Lineam ordinis* n *tranſeuntem per polum C, & concurſus S per Lineam ordinis m; atque in utroque caſu reliquus concurſus O deſcribet Lineam ordinis* 2 n m − m. Vid. fig. Prop. 7, & 8. Demonſtratur ad modum ſuperioris.

PROP.

PROP. X.

Iiſdem poſitis: moveantur concurſus S & N per Lineas datas SAMQ, NECL ordinum n & m, *quarum una SAMQ tranſeat per polum A vel B, altera vero NECL per polum A vel C, ita ut ſimul non tranſeant Lineæ SAMQ, NECL per eundem polum A; dico concurſum O deſcripturum Lineam ordinis* 2nm−m−n.

In plano Curvæ deſcribendæ motu puncti O agatur recta OTZ, & inquirendum ſit in quot punctis Curvam ſecet. Si ducatur concurſus O per rectam OTZ & concurſus S rectarum ASN, BSO per Lineam SAMQ ordinis n tranſeuntem per polum A vel B; reliquus N rectarum ASN, CNO deſcribet Lineam HFCK ordinis $2n-1$, quæ



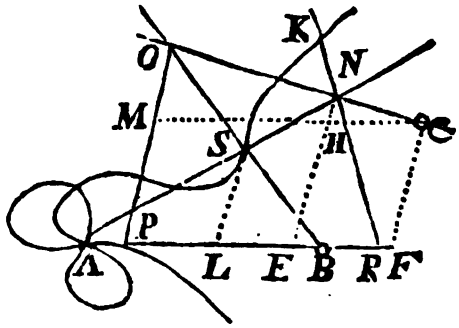
puncta n habeat in polo C (per Corol 2. Prop. 6.) atque Lineæ datæ NECL ordinis m occurrere poteſt in punctis H, F, E, K, L, &c. quorum numerus eſt $2nm-m$. Et viciſſim ſi agatur concurſus N per Lineam HFCK jam deſcriptam ordinis $2n-1$, & concurſus S per Li-

neam SAMQ ordinis *n*; punctum O deſcribet ipſam rectam OTZ Moveantur nunc concurſus N & S per datas Lineas NECL, SAMQ ordinum *m* & *n*, quarum una NECL ordinis *m* tranſeat per polum A vel C, & altera SAMQ ordinis *n* per polum A vel B, ita ut ſimul non tranſeant Lineæ datæ NECL, SAMQ per eundem polum A; & quoties concurſus N percurrendo Lineam datam NECL ordinis *m* occurrat interſectionibus, H, F, E, K, L, *&c.* Lineæ datæ NECL & Lineæ HFCK ordinis $2n-1$, punctum O locabitur in recta OTZ; excepto tantum caſu cum concurſus N, pertranſeat polum C, ibi enim recta CNO erit tangens Curvæ datæ NECL, ac proinde punctum O non erit in recta OTZ. Sed coincidunt *n* puncta interſectionum Linearum NECL, HFCK ordinum *m* & $2n-1$ in polo C; & ideo cum concurſus N occurrat *n* punctis interſectionum in polo C, concurſus O non erit in recta OTZ, adeoque punctum O toties tantum occurret rectæ OTZ quot ſunt unitates in numero $2nm-m-n$: Et inde Linea OWX motu puncti O deſcripta erit ordinis $2nm-m-n$.

PROP. XI.

Moveatur concurſus S rectarum ASN, BSO, per Lineam ASK ordinis n *quæ puncta* r *habeat in alterutro e polis A, B, & concurſus N rectarum ASN, CNO per rectam RN poſitione datam; vel ducatur concurſus N per Lineam ordinis* n *punctaque* r *habentem in alterutro e polis A, C, & concurſus S per rectam poſitione datam; atque reliquus O deſcribet Lineam ordinis* $2n-r$.

Imprimis percurrat concurſus S Lineam ASK ordinis *n* quæ puncta *r* habeat in polo A. Agatur recta BA per polos A, B quæ rectæ NR occurrat in R; deinde ex puncto deſcribente O acta ordinata OP ad rectam AB in dato angulo, ducantur per puncta C, N, S, rectæ CF, NE, SL, ordinatæ OP parallelæ rectamque AB ſecantes in F, E, L: & per polum C ſit CM rectæ AB parallela, rectis NE, OP, occurrens in



in H, M. Ponatur $AB = a$, $AR = b$, $AF = f$, $CF = c$, Curvæ datæ ASK ordinata $SL = s$, abſciſſa $AL = z$, Curvæ deſcribendæ ordinata $OP = y$, abſciſſa $AP = x$, & ob datos angulos trianguli NER fiat $RE : EN :: b : g$. His poſitis eodem calculo quo uſi ſumus in Prop. 4. emerget

$$z = \frac{ayb \times \overline{xg + yf - gf - xc}}{\overline{a-x} \times g \times \overline{yb - yf - bc + xc} + by \times \overline{xg + yf - fg - xc}}$$

& $S = \dfrac{ayg \times \overline{by - yf - cb + xc}}{\overline{a-x} \times g \times \overline{yb - yf - bc + xc} + by \times \overline{xg + yf - fg - xc}}$. Porro cum Curva ASK habeat r puncta in polo A, patet tot eſſe radices tam obſciſſæ z quam ordinatæ S, in æquatione generali, æquales nihilo, quot ſunt unitates in numero r; atque adeo Æquatio erit hujus formæ

$$A z^{r} + B z^{r+1} + C z^{r+2} + D z^{r+3} \ldots\ldots\ldots E z^{n}$$
$$+ S \times \overline{F z^{r-1} + G z^{r} + H z^{r+1} + I z^{r+1} \ldots\ldots\ldots K z^{n-1}}$$
$$+ S^{2} \times \overline{L z^{r-2} + M z^{r-1} + N z^{r} + P z^{r-1} \ldots\ldots\ldots Q z^{n-2}}$$
$$+ S^{3} \times \overline{K z^{r-3} + T z^{r-2} + W z^{r-1} + V z^{r} \ldots\ldots\ldots X z^{n-3}}$$

$+$ &c. $= o$, ubi continuari debet ſeries uſque dum index ordinatæ S fuerit æqualis numero n. Et ideo ſi eliminentur quantitates z & s ſubſtituendo earum valores jam inventos, producetur Æquatio in qua ordinata Curvæ deſcribendæ y & abſciſſa x aſcendent ad $2n$ dimenſiones. Sed nullus erit terminus qui non ducetur in y^{r}. Ergo dividantur termini per y^{r}, & prodibit Æquatio quæ aſcendet ad dimenſiones $2n - r$, atque Linea motu puncti O deſcripta erit ejuſdem ordinis $2n - r$. Et haud abſimili ratiocinio ſi Curva data ASK haberet r puncta in alteru-

tro e polis B, C, & per eam ducatur concurſus S vel N, concluditur Lineam deſcriptam eſſe ordinis $2n - r$.

COROL. I.

Curva deſcripta habere poteſt $n - r$ puncta in dato R. Quippe Curva ASK rectæ AB occurrere poteſt in n punctis & dum concurſus S movendo per Curvam tranſeat per interſectiones rectæ AB & Curvæ ASK; coincident rectæ ASN & BSO, cum BA & punctum deſcribens O locabitur in R; ſi caſus excipias quibus concurſus S perveniat ad r puncta in polo A, nam inibi recta ASN erit tangens Curvæ ASK, & proinde Curva habebit tantum $n - r$ puncta in R. Ducatur autem concurſus N per Curvam ASK & concurſus S per rectam poſitione datam, agaturque recta per polos A, C quæ poſitione datæ occurrat in Q ac eadem ratione demonſtratur Curvam deſcripta habere $n - r$ puncta in Q.

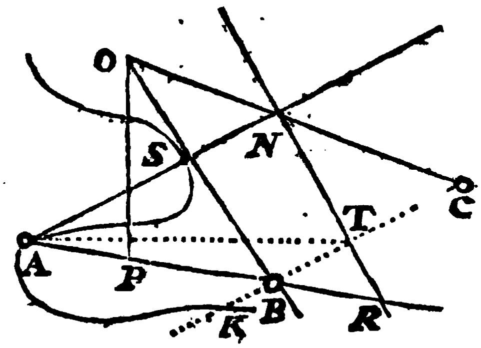
COROL. II.

Curvæ arcus n concurrere poſſunt in polo C. Etenim ducatur recta per polos B, C quæ curvam datam ASK ſecare poteſt in punctis n; & quoties concurſus S movendo attingat interſectiones Curvæ ASK & rectæ BC coincidet recta BSO cum BC ac punctum deſcribens O tranſibit per polum C. Et viciſſim ſi moveatur concurſus N per Curvam datam ASK, concurſus vero S per rectam poſitione datam, Curva deſcripta habere poteſt arcus n in polo B.

COROL. III.

Curva etiam habere poteſt $n - r$ puncta in polo B. Quippe agatur recta per polos B, C, quæ poſitione datæ occurrat in T, junganturque A & T. Et patet cum Curva ASK rectam AT ſecet in n punctis concurſus S movendo attinget rectam AT in totidem punctis, atque coincident

incident rectæ A S N & A T, coeuntibus N & T, niſi caſus excipias quibus concurſus S tranſeat per puncta r in polo A; nam recta ASN ibidem erit tangens Curvæ datæ ASK, & proinde coibunt toties tantum puncta N, T, quot ſunt unitates in numero $n-r$. Cum vero coeant puncta N, T, punctum deſcribens O erit in polo B. Et ſimiliter ratiocinaberis de polo C, ſi concurſus N ducatur per Curvam datam A S K.



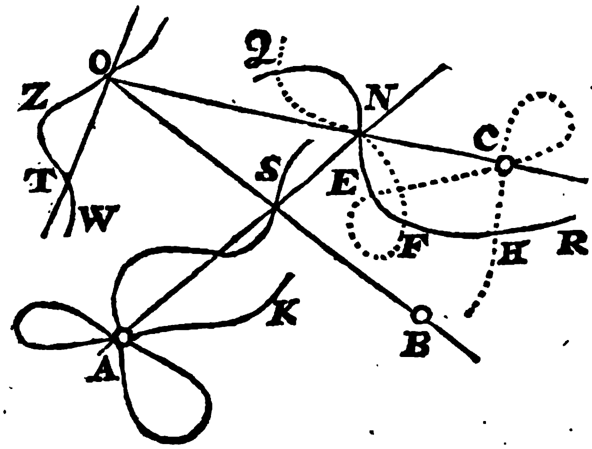
PROP. XII.

Moveatur concurſus S rectarum ASN, BSO, per Lineam SAK, ordinis n, *quæ puncta* r *habeat in alterutro e polis A, B, & concurſus N rectarum ASN, CNO per Lineam NFHR, ordinis* m; *vel moveatur concurſus N quæ puncta* r *habeat in alterutro e polis A, C, & alter S per Lineam ordinis* m; *atque reliquus O deſcribet Lineam ordinis* 2 n m − r m.

* Ut ordo Curvæ innoteſcat, agatur ex puncto deſcribente O recta TO, & quæratur in quot punctis Curva deſcripta huic rectæ occurrere poſſit. Si concurſus O ducatur per rectam T O, & concurſus S per Lineam S A K ordinis n puncta r habentem in polo A aut B, reliquus N deſcribet, per Prop. præced. Lineam QNEC ordinis $2n-r$, quæ Lineæ datæ N F H R ordinis m occurrere poteſt in Q, E, F, H, &c. quorum numerus eſt $2nm-rm$. Et vice verſa ſi concurſus N ducatur per Lineam QNEC jam deſcriptam ordinis $2n-r$, & S per Lineam S A K ordinis n; concurſus O percurret rectam T O. Moveatur nunc concurſus N per Lineam datam N F H R ordinis m, concurſus vero S per

* Vid. fig. ſeq. pag.

per Lineam SAK ordinis n, & quando concurſus N movendo occurrat interſectionibus Q, E, F, H, Linearum NFHR & QNEC ordinum m & $2n-r$ concurſus O tunc erit in recta TO; at cum N ſit in aliis Curvæ NFHR punctis concurſus O non erit in eadem recta



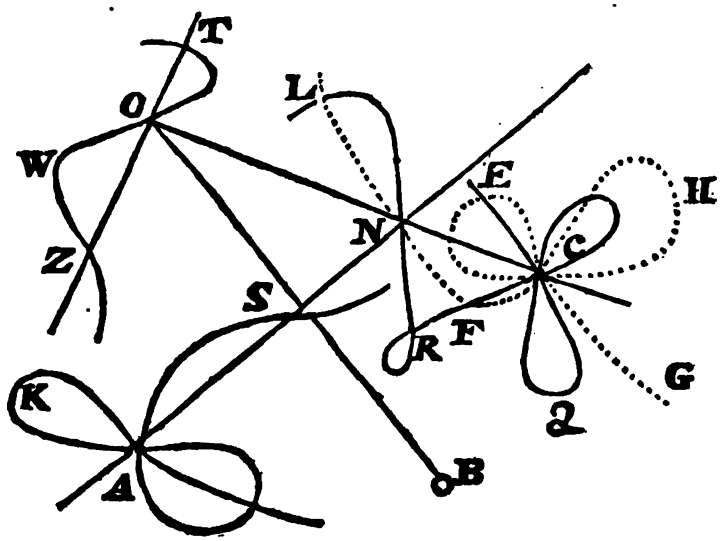
TO, ſed Curvam OZW deſcribet. Et ideo concurſus O toties rectam TO attingere poteſt, quot ſunt unitates in numero $2nm-rm$: atque ita Linea motu puncti O deſcripta erit ordinis $2nm-rm$.

PROP. XIII.

Moveatur concurſus S per Lineam datam AKS ordinis n *quæ habeat* r *puncta in alterutro e polis A, B, & concurſus N per Lineam datam NRCQE ordinis* m *& quæ puncta S habeat in interutro e polis A, C, ita ut Lineæ datæ KAS & NRCQE ſimul non tranſeant per polum A; dico reliquum concurſum O Lineam ordinis* $2nm-rm-sn$ *deſcribere.*

Ponamus Lineam KAS ordinis n puncta r habere in polo A. Agatur recta TOZ, & inveſtigare oportet, quoties Linea deſcripta motu puncti O huic rectæ occurrere poterit. Si concurſus S percurrat Lineam KAS ordinis n quæ habeat r puncta in polo A, concurſus vero O rectam TOZ, reliquus N deſcribet Lineam LFHCG, per Prop. præced. ordinis

dinis $2n-r$ quæ puncta n habere poteſt in polo C, ac Lineæ datæ NRCQE occurrat in punctis L, N, F, C, E, &c. quorum numerus eſt $2nm-rm$. Et viciſſim ſi concurſus N ducatur per Lineam jam deſcriptam LFHCG ordinis $2n-r$, & alter S per Lineam KAS ordinis n, concurſus O percurret rectam TOZ. Moveatur concurſus N per Lineam datam NRCQE ordinis m, concurſus vero S per Lineam KAS ordinis n, & quoties concurſus N movendo tranſeat per interſectiones



L, N, F, C, G, Linearum NRCQE & LFHCG punctum deſcribens O locabitur in recta TOZ; atque dum concurſus N ſit in aliis punctis Lineæ datæ NRQCE ordinis m punctum O non erit in recta TOZ. Sed e punctis interſectionum L, N, F, C, E, &c. quæ ad ſummum ſunt $2nm-rm$, tot coincidunt in polo C quot ſunt unitates in numero sn; nam quia Linea LFHCG ordinis $2n-r$ ibi habeat n puncta, manifeſtum eſt ſi Linea data NRCQE ſemel tranſiret per polum C fient inibi n puncta interſectionum, & ſi bis, ter, quater, &c. tranſiret per polum C, ibidem coibunt $2n$, $3n$, $4n$, &c. puncta interſectionum; atque adeo cum Linea data NRCQE ordinis m toties tranſeat per polum C quot ſunt unitates in s, facile colligitur eſſe ns in-

ter-

terſectiones Curvarum NRCQE & LFHCG in polo C. Cæterum quando concurſus N, movendo per Lineam datam NRCQE ordinis *m*, tranſeat per polum C recta CNO ibidem erit tangens & ideo punctum deſcribens O non erit in recta TOZ, niſi eadem recta CNO fuerit tangens communis ad Curvas NRCQE & LFHCG; atque ita cum concurſus N tranſeat per *sn* puncta interſectionum quæ ſunt in polo C, recta CNO toties erit tangens ad Curvam datam NRCQE. Igitur quum ſint ad ſummum $2nm - rm$ interſectiones Curvarum NRCQE & LFHCG quarum *sn* coincidunt in polo C, patet punctum deſcribens O toties tantum TOZ decuſſare quot ſunt unitates in $2nm - rm - ns$. Ergo Linea motu puncti O deſcripta erit ordinis $2nm - rm - sn$. Et ita ratiocinari licet cum Curva KAS habeat *r* puncta in polo B vel cum Curva NRCQE habeat *s* puncta in polo A.

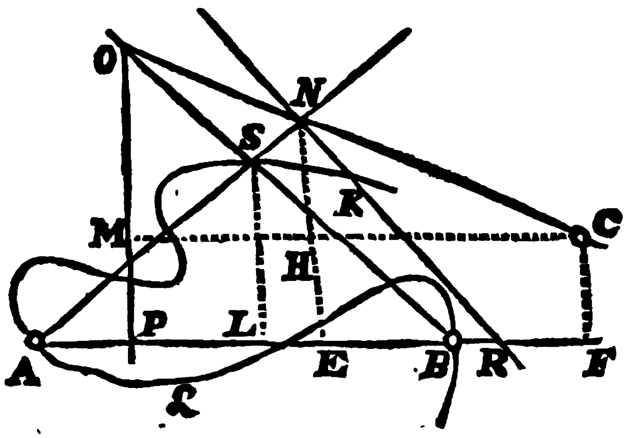
PROP. XIV.

Moveatur concurſus S rectarum ASN, BSO per Lineam datam BQASK, ordinis n, *per polos A & B tranſeuntem, & concurſus N rectarum ASN, CNO per rectam RK poſitione datam; vel ducatur concurſus N per Lineam ordinis* n, *quæ tranſeat per polos A, C, concurſus vero S per rectam poſitione datam; & in utroque caſu punctum O deſcribet Lineam ordinis* $2n - 2$.

Ducatur recta ABF per polos A, B quæ rectæ poſitione datæ KR occurrat in R, & ex puncto deſcribente O ſit ordinata OP in dato angulo ad rectam ABF; atque per puncta C, N, S, ductis CF, NE, SL ordinatæ OP parallelis & rectam ABF ſecantibus in F, E, L, agatur recta CMH per polum C quæ rectis OP, NE, conveniat in M, H. Ponamus $AB = a$, $AR = b$, $AF = f$, $CF = c$, abſciſſam Curvæ deſcribendæ $AP = x$, ordinatam $OP = y$, abſciſſam Curvæ datæ $AL = z$, ordinatam $SL = s$: & ob datos angulos ERN, NER, ſiat $RE : NE :: b : g$. Et eadem analyſi ut in Prop. 4. prodibit

$s =$

$s = \frac{ayg \times \overline{by - yf - cb + xc}}{\overline{a-x} \times g \times \overline{yb - yf - bc + xc} + by \times \overline{xg + fy - fg - xc}}$, & $z = \frac{ayb \times \overline{xg + yf - gf - xc}}{\overline{a-x} \times g \times \overline{yb - yf - bc + xc} + by \times \overline{xg + fy - fg - xc}}$; unde emerget $a - z = \frac{\overline{a-x} \times ag \times \overline{yb - yf - bc + xc}}{\overline{a-x} \times g \times \overline{yb - yf - bc + xc} + by \times \overline{xg + yf - fg - xc}}$. Cæterum quia Curva KSAQB tranſit per polos A, B, & ejus abſciſſa z, incipita polo A, manifeſtum erit, cum evaneſcat ordinata SL $= s$,



unam radicem æquationis reſiduæ æquari nihilo, & alteram eſſe AB $= a$, & proinde termini omnes in æquatione generali in quibus non comparet ordinata s duci debent in $a - z$; atque adeo æquatio generalis Curvæ datæ KSAQB ordinis n erit hujus formæ

$$\overline{Az + Bz^2 + Cz^3 \ldots Dz^{n-1}} \times \overline{a-z} + S \times \overline{E + Fz + Gz^2 \ldots Hz^{n-1}}$$
$$+ S^2 \times \overline{K + Lz + Mz^2 \ldots Nz^{n-2}} + S^3 \times \overline{P + Qz + Rz^2 \ldots Tz^{n-3}}.$$

$+$ &c. $= 0$. Eliminentur autem abſciſſa z & ordinata s, ſubſtituendo earum valores jam inventos quantitatibus x & y involutos, ac producetur æquatio in qua ordinata y & abſciſſa x Curvæ deſcribendæ aſcendent ad dimenſiones $2n$. Sed terminos in ordinem redigendo, conſtabit hanc æquationem dividi poſſe per $ayg \times \overline{by - yf - cb + xc}$; & ideo ſi

dividatur æquatio, aſcendet tantum ad dimenſiones $2n-2$, ac Linea motu puncti O deſcripta erit ordinis $2n-2$. Atque idem demonſtratur cum Curva data K S A Q B tranſeat per polos A, C.

COROL. I.

Curva deſcripta toties tranſire poteſt per utrumque polum B & C quot ſunt unitates in numero $n-1$. Eodem prorſus argumento colligitur ut in Corol. 2 & 3. Prop. 6 & 7.

COROL. II.

Si Curva data K S A Q B ſit Sectio Conica, figura deſcripta erit itidem Sectio Conica.

PROP. XV.

Moveatur concurſus S per Lineam datam B Q A S ordinis n, *quæ habeat puncta* r *in polo A, & p puncta in B, & concurſus N per rectam R N poſitione datam; vel ducatur concurſus N per Lineam ordinis* n, *quæ puncta* r *&* p *habeat in polis A & C, & alter S per rectam poſitione datam; atque reliquus O deſcribet Lineam ordinis* 2 n – r – p.

Conſtruatur figura ut in Prop. præced. atque iiſdem ſymbolis utendo

prodit $AL = z = \dfrac{ayb \times \overline{xg + yf - gf - xc}}{\overline{a-x} \times g \times \overline{yb - yf - bc + xc} + by \times \overline{xg + yf - fg - xc}}$

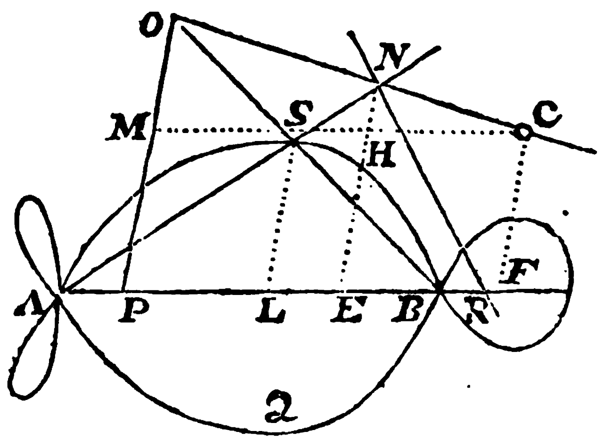
& $SL = s = \dfrac{ayg \times \overline{by - yf - cb + xc}}{\overline{a-x} \times g \times \overline{yb - yf - bc + xc} + by \times \overline{xg + yf - fg - xc}}$

& proinde $a - z = \dfrac{\overline{a-x} \times ag \times \overline{by - yf - cb + xc}}{\overline{a-x} \times g \times \overline{yb - yf - bc + xc} + by \times \overline{gx \times yf - fg - xc}}$.

Sed quia Curva data B Q A S habet puncta r & p, in polis A, B, conſtat, cum evaneſcat ordinata $SL = s$ in æquatione generali, reſiduam tot habere radices nihilo æquales quot ſunt unitates in r, atque etiam tot æ-

quales

quales rectæ $AB=a$, quot ſunt unitates in p: & inde colligitur terminos in quibus non comparet ordinata S ad minimum aſſurgere ad z^r, ac dividi poſſe per poteſtatem $\overline{a-z}|^p$, & terminos in quibus ordinata S ad unicam tantum dimenſionem aſcendit dividi poſſe per $\overline{a-z}|^{p-1}$, eos vero in quibus ordinata S ad duas dimenſiones aſcendit dividi poſſe per

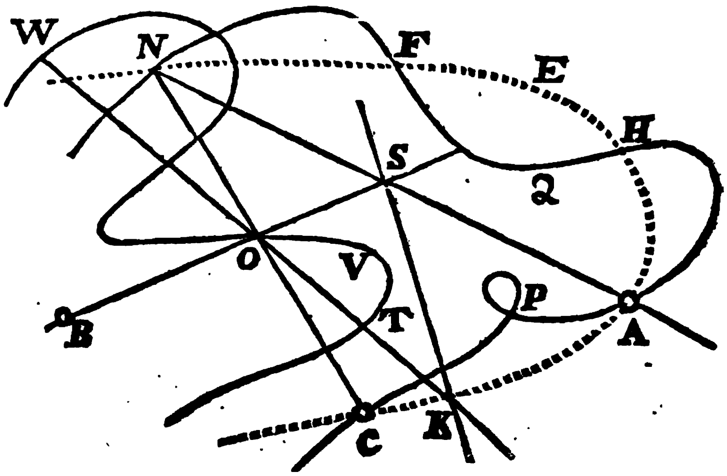


$\overline{a-z}|^{p-2}$ & ita de cæteris. Adeoque æquatio Curvæ datæ BQAS ordinis n erit ut ſequitur. $\overline{Az^r + Bz^{r+1} + Cz^{r+2} \ldots\ldots Dz^{n-p}} \times \overline{a-z}|^p$ $+ S \times \overline{Ez^r + Fz^{r+1} + Gz^{r+2} \ldots\ldots\ldots Hz^{n-p}} \times \overline{a-z}|^{p-1}$ $+ S^2 \times \overline{Kz^r Lz^{r+2} + Mz^{r+2} \ldots\ldots\ldots Nz^{n-p}} \times \overline{a-z}|^{p-2}$ $+ \ldots\ldots + S^p \times \overline{Pz^r + Qz^{r+1} + Rz^{r+2} \ldots\ldots\ldots Tz^{n-p}}$ $+ S^{p+1} \times \overline{Vz^{r-1} + Wz^r + Xz^{r+1} \ldots\ldots\ldots Yz^{n-p-1}} + \&c. = 0$. Igitur ſubſtituendo valores jam inventos ordinatæ S & abſciſſa z, producetur Æquatio in qua Curvæ deſcribendæ ordinata y & abſciſſa x aſcendent ad $2n$ dimenſiones: terminis autem rite diſpoſitis dividi poteſt hæc producta per $y^r \times a^p g^p \times \overline{by-yf-cb+xd}|^p$. Ideoque dividendo per $y^r \times a^p g^p \times \overline{by-yf-cb \times xd}|^p$ proveniet Æquatio in qua x & y aſcen-

y aſcendent tantum ad $2n-r-p$ dimenſiones; atque adeo Linea motu puncti O deſcripta erit ordinis $2n-r-p$. Et ita concludi licet cum concurſus N ducatur per Curvam datam quæ habeat r & p puncta in polis A & C.

Idem Aliter.

Percurrat concurſus S rectam SK poſitione datam, & concurſus N Curvam datam NQAPC quæ habeat r puncta in polo A & p puncta in polo C. Agatur recta TOW & quæratur in quot punctis Curvam deſcriptam ſecet. Conſtat ex Prop. 1. ſi concurſus O & S moverentur



per rectas TOW & SK reliquum N Sectionem Conicam FEAKC deſcribere, quæ tranſibit per polos A, C, & Curvæ datæ ordinis n occurret in punctis N, F, H, A, C, &c. quæ ad ſummum ſunt $2n$. Adeoque viciſſim concluditur ſi concurſus S duceretur per rectam SK & interea concurſus N per Sectionem Conicam FEAKC jam deſcriptam, punctum O ipſam rectam TOW percurrere. Ducatur nunc concurſus N per Lineam datam NQAPC, ordinis n & S per rectam SK. Et quando concurſus N movendo pervenit ad interſectiones N, F, H, A, C, &c. Lineæ NQAPC & Sectionis Conicæ FEAKC, punctum O locabitur in recta TOW; ſi caſus excipias quibus concurſus N tranſeat per polos A, C, etenim dum in iis ſit concurſus N, rectæ ASN, CNK, per

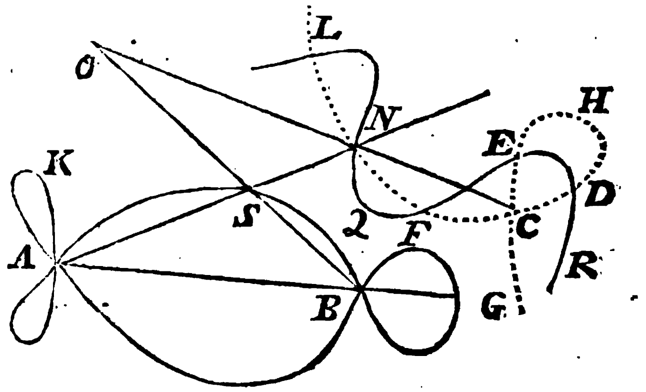
per vices fient tangentes Curvæ NQAPC. Sed Curva data NQAPC habet (ex hyp.) r puncta in polo A & p puncta in polo C; et ideo coincident r interſectiones Curvæ NQAPC cum Sectione Conica in polo A, & p interſectiones in polo C. Igitur cum concurſus N tranſeat per r interſectiones in polo A & p interſectiones in polo C, toties rectæ ASN, CNO per vices erunt tangentes Curvæ NQAPC, & punctum O decedet a recta TOW: & ita cum concurſus N perveniat ad $2n-r-p$ interſectiones Lineæ NQAPC & Sectionis Conicæ FEAKC, concurſus O rectam TOW in totidem punctis tantum attinget. Ergo Linea motu puncti O deſcripta erit ordinis $2n-r-p$.

COROL.

Si Curva data habeat r puncta in polo A, & p puncta in B Linea deſcripta habebit $n-r$ puncta in B, & $n-p$ puncta in C: & viciſſim ſi Curva data habeat r puncta in polo A, & p puncta in B, Linea deſcripta habebit $n-r$ puncta in B, & $n-p$ puncta in C. Demonſtratur ad modum Corol. 2 & 3, Prop. 6 & 7.

PROP. XVI.

Iiſdem poſitis: ducatur concurſus S per Lineam AKSB ordinis n, *quæ puncta* r *habeat in polo A, &* p *puncta in B, concurſus vero N per Lineam LNQER ordinis* m; *vel percurrat concurſus N Lineam ordinis* n, *quæ habeat* r *puncta in A, &* p *puncta in C, & concurſus S Lineam ordinis* m; *dico reliquum O Lineam ordinis* 2nm − rm − pm *deſcribere.*

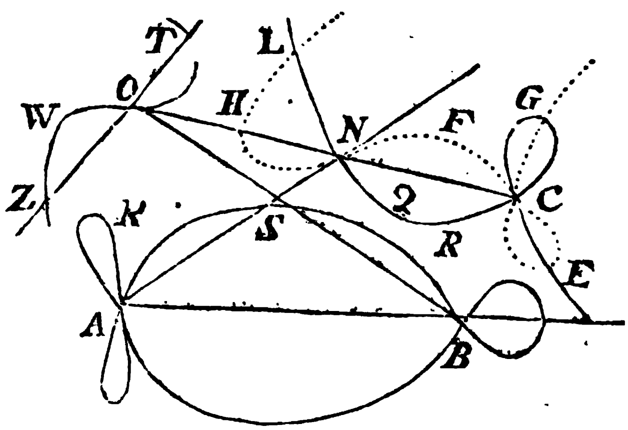


Demonſtratur ad modum Prop. 12.

PROP. XVII.

Ducatur concurſus S per Lineam AKSB ordinis n, *quæ habeat* r *puncta in polo A,* & p *puncta in B,* & *concurſus N per Lineam LNQRCE ordinis* m *quæ habeat* s *puncta in polo C; vel percurrat concurſus N Lineam, ordinis* n *quæ habeat* r *puncta in A,* & p *puncta in C,* & *concurſus S Lineam ordinis* m, *quæ habeat* s *puncta in B; dico punctum O deſcripturum Lineam ordinis* $2nm-rm-pm-sn+sp$.

Agatur recta TOZ, & inveſtigandum ſit quoties Curvæ motu puncti O deſcriptæ occurrere poteſt. Si concurſus S moveatur per Lineam datam AKSB ordinis n, quæ habeat r puncta in polo A, & p puncta in B, & interea concurſus O per rectam datam TOZ; punctum N deſcribet, per Prop. 15. Lineam LHNFCG ordinis $2n-r-p$, quæ



puncta $n-p$ habebit in polo C, & Curvæ alteri datæ LNQRCE ordinis m occurret in punctis L, N, C, E, G, &c. quæ ad ſummum ſunt $2nm-rm-pm$; & contra ſi concurſus N percurrat Lineam LHNFCG jam deſcriptam, concurſus vero S Lineam datam AKSB ordinis n, punctum O in recta TOZ movebitur. Ducatur nunc concurſus N per Lineam datam LNQRCE, ordinis m, & quoties movendo perveniat ad interſectiones L, N, C, E, G, &c. Linearum LNQRCE, LHNFCG ordinum m & $2n-r-p$, quarum numerus $2nm-rm-pm$,

punctum

punctum deſcribens O attinget rectam TOZ; niſi caſus excipias quibus concurſus N tranſeat per polum C, ibi enim recta CNO fiet tangens, & punctum O decedet a recta TOZ. Sed cum Linea LNQRCE ordinis m habent s puncta in polo C, & Linea LHNFCG ordinis $2n-r-p$ ibidem habeat $n-p$ puncta, coincident $sn-sp$ interſectiones in polo C; & proinde punctum O Curvam TOWZ deſcribendo toties tantum rectam TOZ decuſſabit quot ſunt unitates in $2nm-rm-pm-sn+sp$. Ergo Linea motu puncti O deſcripta erit ordinis $2nm-rm-pm-sn+sp$. Et ſimili ratiocinio de altero caſu in Prop. concluditur.

COROL.

Hinc ſi Lineæ AKSB, LNQRCE, ordinis n & m fuerint Sectiones Conicæ, Linea deſcripta erit tertii ordinis.

PROP. XVIII.

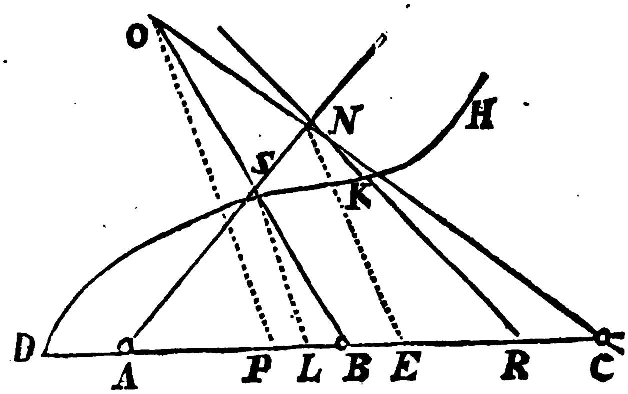
Locentur tres poli A, B, C, in eadem recta ABC, & ducatur concurſus S per Lineam DSH ordinis n, *concurſus vero N per rectam KNR; & reliquus O deſcribet Lineam etiam ordinis* n.

* Ducatur a puncto deſcribente O ordinata OP in dato angulo ad rectam ABC, & ei occurrat in P; deinde per S & N agantur ordinatæ OP, parallelæ SL & NE, rectamque ABC ſecent in L & E. Dicatur $AB=a$, $AR=b$, $AC=c$, $AE=u$, abſciſſa Curvæ datæ $AL=z$, ejus ordinatæ $SL=s$, abſciſſa Curvæ deſcribendæ $AP=x$, ordinata $OP=y$; & propter triangulum NER ſpecie datum ſit $RE : NE :: b : g$, & inde $NE=\frac{bg-gu}{b}$. Quoniam triangula ASN, ANE, ſunt æquiangula & $AL : LS :: AE : NE$ erit $z : s :: u : \frac{bg-gu}{b}$; & ideo $u=\frac{zbg}{bs+zg}$. Et ſimiliter ob triangula CNE, COP æquiangula, ac $CE : EN :: CP : PO$. Prodit $c-u : \frac{bg-gu}{b} :: c-x : y$; & proinde $\frac{bcy-bcg+bgx}{by-cg+xg}=u$: unde

zby

* Vid. fig. ſeq. pag.

$\frac{zbg}{bs+zg} = \frac{bcy-bcg+bgx}{by-cg+xg}$, & reducendo provenit $s = \frac{byzg-zcyg}{bcy-bcg+bgx}$. Porro qui triangula BLS, BPO ſunt ſimilia & BL : LS :: BP : PO erit $a-z : s :: a-x : y$, & propterea $s = \frac{\overline{a-z}\times y}{a-x}$. Æquentur va-

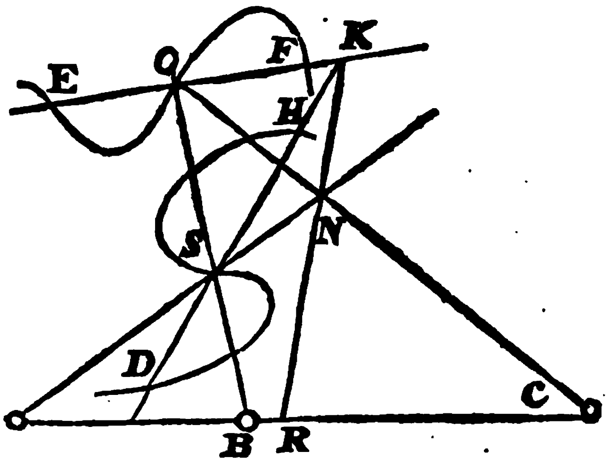


lores rectæ SL $= s$, & habebitur $z = \frac{ab\times\overline{cy-cg+gx}}{abg-acg+cgx+bcy-bcg}$, & hoc ſubſtituto in $\frac{\overline{a-z}\times y}{a-x}$ fiet $s = \frac{ag\times\overline{b-c}\times y}{abg-acg+cxg+bcy-bcg}$; atque adeo in valoribus rectarum z & s ordinata y & abſciſſa x ad unicam tantum dimenſionem aſcendunt. In æquatione generali Curvæ datæ DSH, $S^n + \overline{Az+B}\times S^{n-1} + \overline{Cz^2+Dz+E}\times S^{n-2} + \overline{Fz^3+Gz^2+Hz+K}\times S^{n-3} + \&c. = 0$, ſubſtituantur valores inventi rectarum z & s, atque producetur Æquatio ubi ordinata y & abſciſſa x erunt ejuſdem ordinis n. Ideoque Linea motu puncti O deſcripta erit ordinis n.

Idem Aliter.

Nam ſi agatur recta EOK in plano Lineæ deſcribendæ, & ducantur concurſus O & N per rectas EOK, RNK, reliquus S per Corol. 15. Prop:

Prop. 1. deſcribet rectam KSD, quæ occurrere poteſt Lineæ datæ HSD in punctis H, S, D, &c. quorum numerus erit *n*: & viciſſim ſi ducantur concurſus S & N per rectas KSD, RKN, reliquus O percurret rectam



EOK. Moveatur jam concurſus S per Lineam datam HSD ordinis *n*, alter vero N per rectam RNK; & quoties concurſus S occurrat interſectionibus H, S, D, &c. rectæ KSD & Lineæ datæ HSD, punctum O erit in recta EOK, at non ſæpius. Ideoque Linea EOF motu puncti O deſcripta occurrere poteſt rectæ EOK in *n* punctis, & propterea erit ordinis *n*.

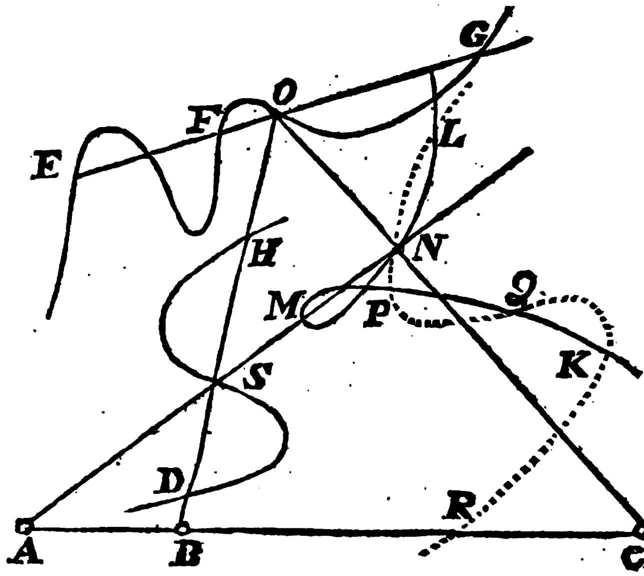
PROP. XIX.

* *Iiſdem poſitis: ducantur concurſus S & N per Lineas DSH & LNMQ, ordinum* n *&* m; *& reliquus O deſcribet Lineam ordinis* nm.

Agatur recta FOG in plano Lineæ deſcribendæ, ac per eam moveatur concurſus O, alter vero S per Lineam HSD ordinis *n*; & reliquus N deſcribet Lineam NQKR etiam ordinis, *n* per Prop. præced. quæ Lineæ datæ LNMQ ordinis *m*, occurrere poteſt in punctis L, N, P, Q, K, &c. quorum numerus eſt *nm*: & contra ſi ducatur concurſus

* Vid. fig. ſeq. pag.

S per Lineam DSH ordinis *n*, & concurſus N per Lineam jam deſcriptam NQKR ejuſdem ordinis *n*; punctum O percurret ipſam rectam FOG. Ducatur nunc concurſus S per Lineam datam DSH ordinis *n*,



ac concurſus N per Lineam datam LNMQ ordinis *m*, & quando punctum N occurrat interſectionibus Linearum LNMQ & NQKR; punctum O erit in recta FOG. Sed cum N ſit in aliis Curvæ LNMQ punctis, punctum O rectam FOG non attinget. Igitur Linea EFOG motu puncti O deſcripta occurrere poteſt rectæ FOG in *nm* punctis, & proinde erit ordinis *nm*.

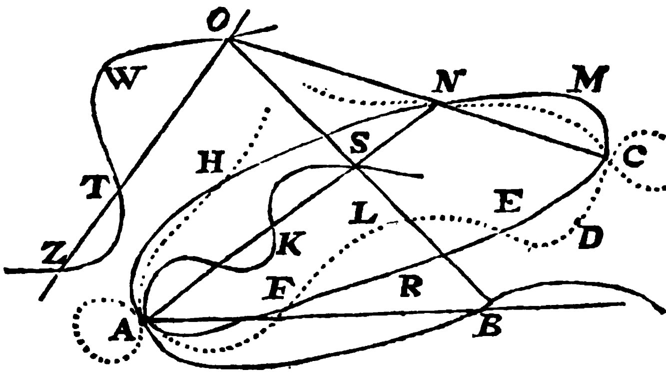
PROP. XVII.

* *Poſitis quæ in Prop.* 14. *Ducatur concurſus S per Lineam SKAB ordinis* n, *quæ ſemel tranſeat per polos A, B, & concurſus N per Lineam NMCRA ordinis* m, *ſemel tranſeuntem per polos A, C; dico reliquum concurſum O deſcripturum Lineam ordinis* 2nm − 2m − 2n + 2.

Agatur recta OTZ, & quæratur in quot punctis Linea motu puncti O deſcripta huic rectæ occurrere poteſt. Si moveretur concurſus O per rectam.

* Hæc Prop. collocari debuit inter 14 & 15, ſed currente prelo omiſſa fuit.

rectam OTZ, & interea concurſus S per Lineam SKAB ordinis n, reliquus N deſcribet Lineam NCDLA ordinis $2n-2$, per Prop. 14, quæ puncta $n-1$ habebit in utroque polo A, C, & conveniet Lineæ NMCRA ordinis m in punctis N, C, E, F, A, H, &c. quæ ad ſummum ſunt $2nm-2m$: & vice verſa ſi ducatur concurſus N per ipſam Lineam NCDLA jam deſcriptam & interea concurſus S per Lineam SKAB, punctum O percurret rectam OTZ. Adeoque quoties concurſus N movendo per Lineam NMCRA ordinis m perveniat ad interſectiones N, C, E, F, A, H, &c. Linearum NMCRA & NCDLA, punctum O locabitur in recta OTZ, at non ſæpius, exceptis tantum caſibus quando



concurſus N tranſeat per polos A, C, ibi enim rectæ ASN, CNO alternatim fient tangentes Curvæ NMCRA, & punctum O recedet a recta OTZ. Sed coincidunt $2n-2$ interſectiones Linearum NMCRA & NCDLA in polis A, C, & proinde punctum O toties tantum rectam OTZ attinget quot ſunt unitates in numero $2nm-2m-2n+2$. Ergo Linea motu puncti O deſcripta erit ordinis $2nm-2m-2n+2$.

COROL.

Hinc ſi Curvæ SKAB & NMCRA fuerint Sectiones Conicæ, Curva deſcripta evadet etiam Sectio Conica quæ tranſibit per polos B, C.

SECTIO III.

Ubi deſcribuntur Sectiones Conicæ ope plurium rectarum circa polos moventium.

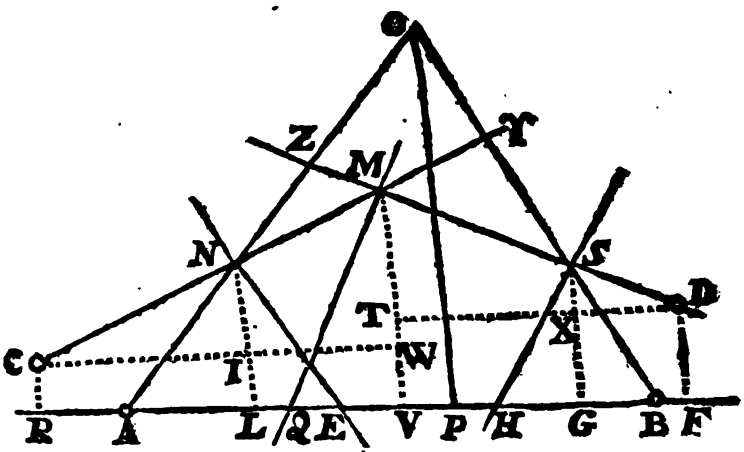
PROP. XXI.

Circa quatuor puncta A, B, C, D, data in plano quovis, tanquam polos, moveantur quatuor rectæ ANO, BSO, CNM, DSM, quæ ſeſe interſecent in ſex punctis M, N, O, S, Z, T; ducantur tria interſectionum puncta M, N, S, per tres rectas MQ, NE, SH poſitione datas; dico reliqua O, T, Z, Sectiones Conicas deſcribere.

Per puncta A & B ducatur recta RABF datis NE, MQ, SH, occurrens in E, Q, & H. Eligatur e tribus O, Y, Z, punctum O, & acta OP ordinata ad rectam AB in dato angulo inclinata, ex punctis C, N, M, S, D, ducantur rectæ CR, NL, MV, SG, DF, ordinatæ OP parallelæ, quæ rectæ RABE occurrant in R, L, V, G, F; & ideo datæ ſunt CR, DF, AB, AE, ER, BH, FQ, RQ, HF, denique ex datis C & D agantur CIW, & DXT rectæ RABF parallelæ, ſecentque rectas NL, MV, & SG in punctis I, W, T, X, & inde erit CR=IL=VW, & DF=GX=VT. Sit AB$=a$, AE$=k$, RE$=r$, BH$=b$, FQ$=f$, CR$=c$, DF$=d$, RQ$=q$, FH$=h$, EL$=z$, QV$=u$, HG$=s$, AP$=x$, PO$=y$. Quoniam poſitione dantur rectæ NE, MQ, HS & AB, dabuntur anguli AEN, BQM, BHS, & quia anguli ALN, AVM, & AGS ſunt æquales dato APO, datæ erunt triangulorum LEN, QMV & SHG rationes laterum. Fiat EL : LN :: $a : n$ & QV : MV :: $a : m$, & HG : GS :: $a : g$: Adeoque erit LN$=\frac{nz}{a}$, MV$=\frac{um}{a}$, GS$=\frac{sg}{a}$. Porro autem quia æquiangula ſunt triangula ALN & AOP & inde AL : LN :: AP : PO erit

$k-z$.

$k-z:\frac{nz}{a}::x:y$, & proinde $nzx=aky-azy$, & $z=\frac{aky}{nx+ay}$. Et ſimiliter ob triangula CIN & CWM æquiangula & CW : WM :: CI : IN erit $q+u:\frac{um}{a}-c::r-z:\frac{nz}{a}-c$ unde $rum-rac-zum+zac=qnz-qca+unz-uac$ & $z=\frac{rum-rac+qca+uac}{um-ac+qn+un}$. Æquentur valores rectæ EL $=z$ & erit $u\times\overline{rm+ac}\times\overline{nx+ay}+\overline{qca-rac}\times\overline{nx+ay}=u\times\overline{m+n}\times aky+\overline{qn-ca}\times aky$ & ideo $u=\frac{\overline{rac-qca}\times\overline{nx+ay}+\overline{qn-ca}\times aky}{\overline{rm+ac}\times\overline{nx+ay}-aky\times\overline{m+n}}$. Triangula BGS & BPO



ſunt etiam æquiangula & BG : GS :: BP : PO, & proinde erit $b-s:\frac{sg}{a}::a-x:y$, & ideo $bya-say=sga-sgx$ & $s=\frac{bya}{ay+ag-gx}$. Denique cum triangula DXS & DTM ſunt ſimilia & DX : XS :: DT : TM erit $h-s:\frac{gs}{a}-d::f-u:\frac{um}{a}-d$ adeoque $hum-had-sum+sad=fgs-fad-gsu+adu$ & $s=\frac{had-hum-fad+adu}{gu-fg-um+ad}$

unde

unde erit $\frac{bya}{ay+ag-gx} = \frac{had-hum-fad+dau}{gu-fg-um+ad}$ & fractiones reducendo $u \times \overline{da-hm} \times \overline{ay+ag-gx} + \overline{had-fad} \times \overline{ay+ag-gx} = u \times \overline{g-m} \times bya + \overline{ad-fg} \times bay$ & inde $u = \frac{\overline{fad-had} \times \overline{ay+ag-gx} + \overline{ad-fg} \times bya}{\overline{da-hm} \times \overline{ay+ag-gx} + \overline{m-g} \times bya}$. Ergo ſi æquentur duo valores rectæ $u =$ QV, jam reperti, prodibit æquatio ad Sectionem Conicam. Eadem analyſi mutatis mutandis patebit puncta reliqua Z & Y Sectiones Conicas deſcribere.

Idem aliter.

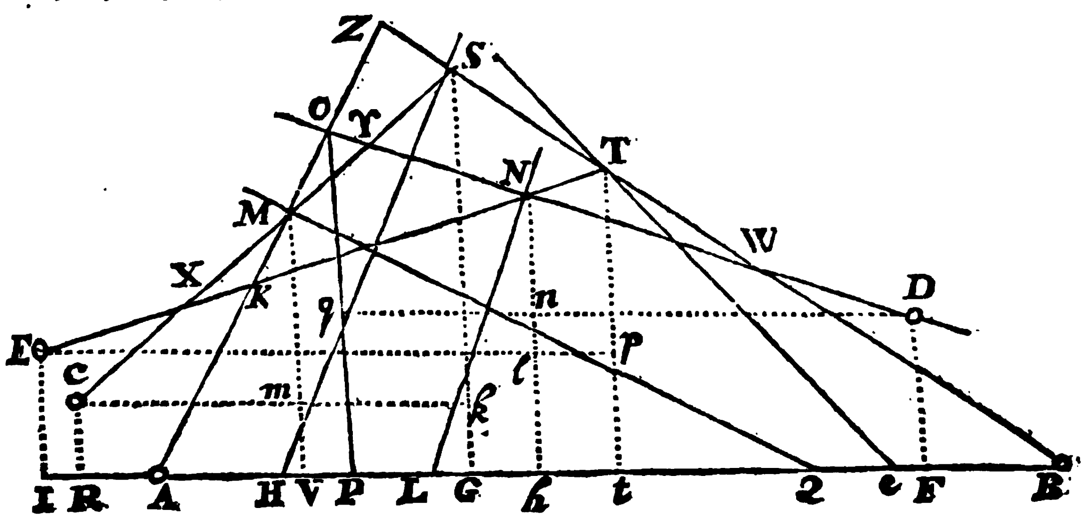
Quoniam circa polos C, B, D moventur tres rectæ CMY, BSY, DSM, ſeſe interſecantes in tribus punctis M, Y, S, quorum duo M & S ducuntur per datas rectas QM & HS, reliquum Y per Prop. 1. Sectionem Conicam deſcribet per polos C & B tranſeuntem. Similiter quia circa polos A, C, D, moventur tres rectæ ANZ, CNM, DMZ ſibi occurrentes in tribus punctis N, Z, M, quorum duo M, N, per datas rectas EN & QM ducuntur, tertium Z etiam Sectionem Conicam deſcribet quæ tranſibit per polos A & D. Cæterum circa polos A, B, D moventur tres rectæ AZO, BSO, DSZ, ſibi occurrentes in tribus punctis Z, O, S, quorum unum S per datam rectam SH ducitur & alterum Z per Sectionem Conicam per polos A & D tranſeuntem; & ideo reliquum O per Corol. Prop. 14. etiam Sectionem Conicam deſcribet. Ergo tria puncta Z, O, Y, Sectiones Conicas deſcribent dum reliqua tria N, M, S per datas rectas EN, QM, SH ducuntur.

PROP. XXII.

Circa quinque puncta data A, B, C, D, E in plano quovis, tanquam polos, moveantur quinque rectæ AKM, BTS, CXS, DNO, ENT, quæ ſeſe interſecent in decem punctis M, N, S, T, K, O, W, X, Z, Y; ducantur quatuor interſectionum puncta M, N, S, T, per quatuor rectas MQ, NL,

S H, T e, poſitione datas; dico ſex reliqua K, O, W, X, Z, Y, Sectiones Conicas deſcribere.

E ſex punctis K, O, W, X, Z, Y, eligatur quodlibet O, rectarum AMO & DNO concurſus & quæratur Æquatio Curvæ deſcriptæ. Per puncta data A & B agatur recta datis SH, LN, MQ, T *e* occurrens in punctis H, L, Q, *e*. Ex puncto deſcribente O ducatur ordinata O P ad rectam A B in dato quovis angulo OPR inclinata atque per puncta C, D, E, M, N, S, T, age rectas CR, DF, EI, MV, N*h*, SG, T*t*, or-



dinatæ OP, parallelas & rectæ A B occurrentes in R, F, I, V, *h*, G, *t*, & ideo datæ ſunt C R, DF, EI, LI, AQ, QR, HR, AF, BH, B*e*, *e*I, FL, denique per puncta C, D, E, ducantur C*mk*, D*nq*, E*lp*, rectæ AB parallelæ & rectis MV, OP, SG, N*h*, T*t* occurrentes in *m*, *q*, *k*, *l*, *n*, *p* & proinde $CR = mV = kG$, $DF = nh = qP$, $EI = lh = pt$. Dicatur $AQ = a$, $Be = b$, $CR = c$, $DF = d$, $EI = e$, $AF = f$, $LI = q$, $QR = r$, $HR = p$, $BR = h$, $eI = i$, $FI = l$, $HG = s$, $et = t$, $QV = u$, $Lh = z$, ordinata $OP = y$, abſciſſa $AP = x$. Quoniam poſitione dantur quatuor rectæ MQ, N L, SH, T*e*, dabuntur anguli MQA, NLB, SHB, T*e*A, & quia anguli MVA, N*h*A, SGA, T*t*A, ſunt æquales dato OPA, datæ erunt rationes laterum triangulorum MQV, NL*h*, SHG, T*te*.

Eſto

Eſto $QV : VM :: a : m$. $Lh : hN :: a : n$. $HG : GS :: a : g$ & $te : Tt :: a : k$; unde erit $VM = \frac{um}{a}$, $hN = \frac{zn}{a}$, $SG = \frac{sg}{a}$, $Tt = \frac{tk}{a}$. Cæterum propter triangula AVM & APO æquiangula, & $AP : PO :: AV : VM$ erit $x : y :: a - u : \frac{um}{a}$ & $u = \frac{ya^2}{ay + xm}$. Et ſimiliter ob triangula CMm & CSk æquiangula, & $Cm : Mm :: Ck : ks$ erit $r - u : \frac{um}{a} - c :: p + s : \frac{sg}{a} - c$, & $u = \frac{pac + sac + sgr - arc}{pm + sm + sg - ac}$, adeoque erit $\frac{ya^2}{ay + xm} = \frac{pac + sac + sgr - arc}{pm + sm + sg - ac}$; unde

$$s = \frac{\overline{rc - pc - ac + pm} \times a^2 y + xm \times \overline{rac - apc}}{-ya^2 \times \overline{m + g} + \overline{ay + xm} \times \overline{ac + gr}}.$$

Sed triangula BTt & BSG ſunt etiam ſimilia, & $Bt : Tt :: BG : GS$. Et ideo erit $b + t : \frac{tk}{a} :: b - s : \frac{sg}{a}$, & $s = \frac{tkb}{bg + tg + tk}$, & proinde

$$\frac{tkb}{bg + tg + tk} = \frac{\overline{rc - pc - ac + pm} \times a^2 y + xm \times \overline{rac - apc}}{\overline{ay + xm} \times \overline{ac + gr} - ya^2 \times \overline{m + g}},$$

& invenitur

$$t = \frac{bga^2 y \times \overline{rc - pc - ac + pm} + cgaxmb \times \overline{r - p}}{kbay \times \overline{ac + gr + am + ag} + kbxm \times \overline{ac + gr + ga} + ka \times \overline{-rcay + pcay + a^2 cy - pmay - xmrc + xmpc}}$$

Quinetiam triangula DNn & DOq ſunt ſimilia, & $Dn : Nn :: Dq : Oq$ erit $l - z : \frac{zn}{a} - d :: f - x : y - d$, & inde $Z = \frac{\overline{ld - ly - df + dx} \times a}{nx - nf - ay + ad}$. Denique ob æquiangula triangula ElN & ETp, & $El : lN :: Ep : pT$, eſt $q + z : \frac{zn}{a} - e :: i - t : \frac{tk}{a} - e$; & inde $z = \frac{qtk - qae + aei - aet}{ni - nt - tk + ae}$, & propterea $\frac{\overline{ld - ly - df + dx} \times a}{nx - nf - ay + ad} = \frac{qtk - qae + aei - aet}{ni - nt - tk + ae}$; unde obtinebitur

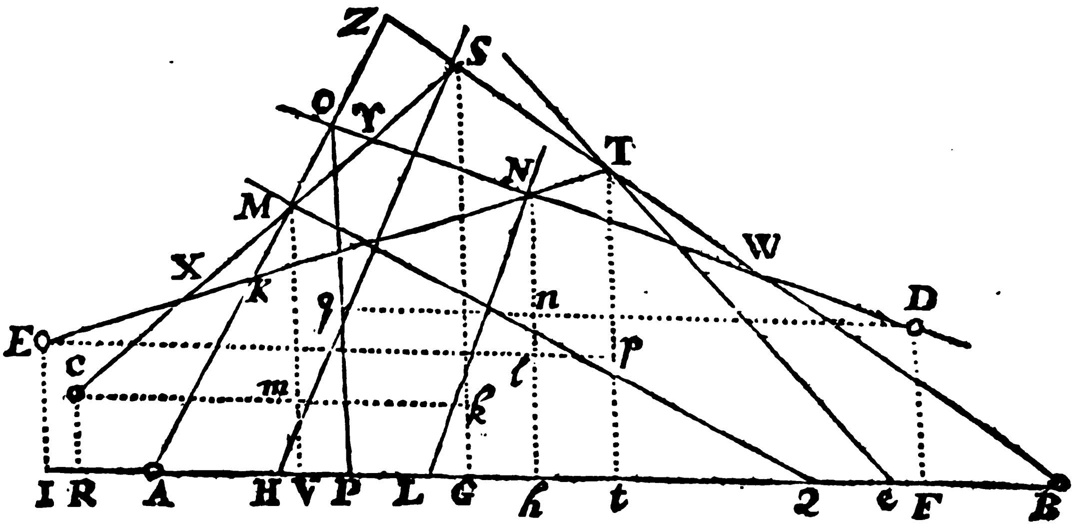
$$t = \frac{a \times \overline{ei - qe} \times \overline{nx - nf - ay + ad} + a \times \overline{ni + ae} \times \overline{ly - ld + df - dx}}{\overline{ly - ld + df - dx} \times a \times \overline{n + k} + \overline{ae - qk} \times \overline{nx - fn - ay + ad}}$$

Æquen-

Æquentur duo valores jam inventi rectæ $te = t$, & denominatorem unius ducendo in numeratorem alterius orietur Æquatio ad Sectionem Conicam. Et ſimili ratiocinio demonſtratur quinque puncta reliqua K, W, Y, X, Z, Sectiones Conicas deſcribere.

Idem Aliter.

Quoniam concurſus N & T rectarum ENT, DWN, BWT, moventur per rectas NL, Te; reliquus W per Prop. 1. deſcribet Sectionem Conicam per polos D, B, tranſeuntem: & quum concurſus S & T ectarum BTS, CXS, EXT percurrunt rectas SH, Te, reliquus X



deſcribet Sectionem Conicam per polos C, E: & ſimiliter quia concurſus S & M rectarum CMS, BSZ, AMZ, ducuntur per rectas SH, MQ; punctum Z deſcribet Sectionem Conicam per polos A, B. Præterea cum concurſus S rectarum BWS, CYS, movetur per rectam SH, & concurſus W rectarum DWY, BWS per Sectionem Conicam polos D, B

tranſeuntem; punctum Y per Corol. Prop. 14. deſcribet Sectionem Conicam quæ attinget polos C, D: & pariter cum concurſus M rectarum CXM, AKM, ducitur per rectam MQ, ac concurſus X rectarum EXK, CXM per Sectionem Conicam attingentem polos E, C; punctum K Sectionem Conicam deſcribet per polos E, A: denique concurſus M rectarum AMO, CMY, movetur per rectam MQ & concurſus Y rectarum DYO, CMY per Sectionem Conicam polos C, D percurrentem; & ideo punctum O deſcribet Sectionem Conicam per polos A, D. Ergo ſex puncta W, X, Z, Y, K, O, Sectiones Conicas deſcribent.

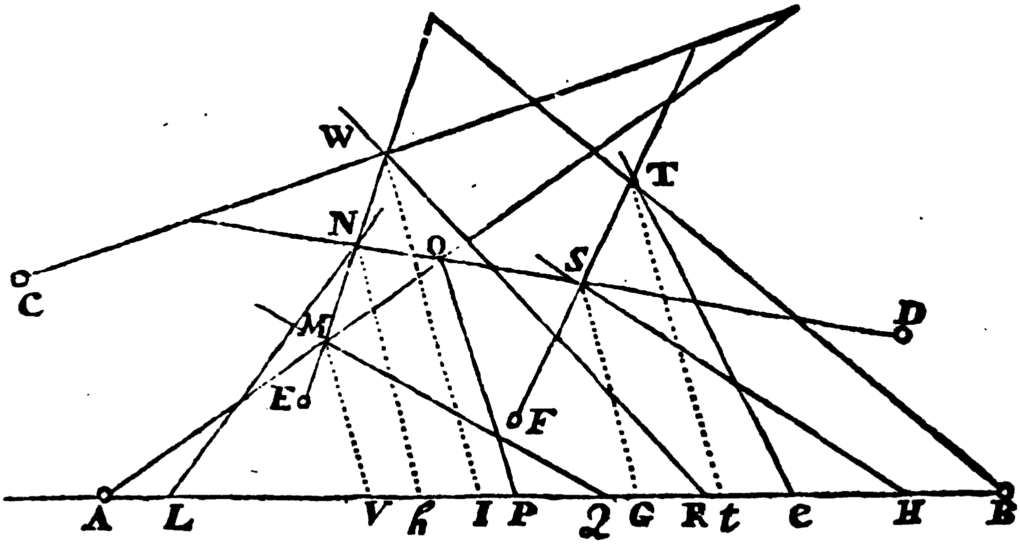
PROP. XXIII.

Iiſdem poſitis ſit n *numerus rectarum circa puncta A, B, C, D, E, F,* &c. *tanquam polos moventium quæ ſeſe interſecent in punctis quæ ad ſummum ſunt* $\frac{n^2-n}{2}$; *ducantur interſectionum puncta* $n-1$ *per rectas totidem datas; dico reliqua* $\frac{n^2-3n+2}{2}$ *Sectiones Conicas deſcribere.*

E punctis $\frac{n^2-3n+2}{2}$ quæ per rectas poſitione datas non ducuntur eligatur quodlibet O & quæratur Æquatio Curvæ deſcriptæ. Per puncta A, B agatur recta AB poſitione datis MQ, NL, SH, T*e*, WR, &c. occurrens in Q, L, H, *e*, R, &c. & a puncto O ſit ordinata OP in dato angulo ad rectam AB, atque per interſectiones M, N, S, T, W, &c. quæ poſitione datas percurrunt agantur ordinatæ OP parallelæ MV, N*h*, SG, T*t*, WI, &c. rectamque AB ſecent in V, *h*, G, *t*, I, &c. Dicatur $AP=x$, $Po=y$, $QV=u$, $Lh=z$, $HG=s$, $et=t$, $IR=r$, &c. Quoniam tot ſunt variabiles u, z, s, t, r, &c. præter ordinatam y & abſciſſam x quot ſunt rectæ poſitione datæ quarum numerus eſt $n-1$;

mani-

manifeſtum eſt ex analyſi duarum Prop. precedentium totidem eſſe æquationes quibus variabiles u, z, s, t, r, &c. determinantur quot ſunt circa polos A, B, C, D, E, F, &c. rectæ mobiles quarum numerus eſt n; & ideo exterminari poſſunt omnes variabiles u, z, s, t, r, &c: Atqui duæ tantum ſunt æquationes in quibus occurrunt ordinata y & abſciſſa x, hujus formæ $Axu + Byu + Cx + Dy = Eu + F$ & $Gxs + Hys + Ix + Ky = Ls + M$, ubi x, y, u, s, ad unicam dimen-



ſionem aſcendunt & coefficientes A, B, C, D, &c, ſunt quantitates cognitæ. Cæteræ vero æquationes ſunt hujuſmodi $Nuz + Pu + Qz + R = o$, $Szr + Tz + Wr + X = o$, $ats + bt + cs + d = o$, $frt + gr + ht + k = o$, &c. ubi quævis ex indeterminatis u, s, z, r, t, eſt unius dimenſionis atque occurrit in duabus tantum æquationibus, & inde valor ejus exprimi poteſt per fractionem in cujus numeratore & denominatore

 occur-

occurrit alia variabilis etiam unius dimenſionis, & hujus iterum valor exprimi poteſt per aliam fractionem in qua eſt alia variabilis unius tantum dimenſionis, & ſic deinceps. Unde ſi inveniantur duo valores cujuſvis ex indeterminatis u, s, z, r, t, poſſunt hi tandem exprimi duabus fractionibus, in quarum numeratoribus & denominatoribus abſciſſa x & ordinata y ad unicam dimenſionem aſcendent. Æquentur valores reperti, & ducendo unius numeratorem in denominatorem alterius, producetur Æquatio ad Sectionem Conicam.

Idem aliter.

CAS. I.

Locentur puncta interſectionum quæ per rectas poſitione datas ducuntur in eadem recta. Dicantur rectæ, quæ circa polos moventur K, L, M, N, P, Q, &c. quarum numerus ſit n; & moveantur interſectiones rectæ K cum cæteris L, M, N, P, Q, &c. per rectas poſitione datas quarum numerus $n-1$. Quoniam numerus interſectionum rectarum K, L, M, N, P, Q, &c. ad ſummum eſt $\frac{n^2-n}{2}$ ſi ſubducatur $n-1$ numerus interſectionum rectæ K cum cæteris L, M, N, P, Q, &c. quæ per poſitione datas ducuntur reſtabit $\frac{n^2-3n+2}{2}$ pro reliquo numero interſectionum rectarum L, M, N, P, Q, &c. quæ Curvas deſcribent. Sed manifeſtum eſt omnes rectas, K, L, M, N, P, Q, &c. tot triangula conficere baſes habentia in eadem recta K quot ſunt unitates in numero $\frac{n^2-3n+2}{2}$; & ideo cum horum omnium anguli qui ſunt ad baſes ducuntur per rectas poſitione datas patet eorum vertices per Prop. 1. Sectiones Conicas deſcribere. Ergo omnes interſectiones quæ rectas

non

non percurrunt quarum numerus $\frac{n^2 - 3n + 2}{2}$ Sectiones Conicas deſcribent.

CAS. II.

Locentur utcunque puncta interſectionum, quæ per rectas poſitione datas ducuntur. Quoniam una ex interſectionibus cujuſvis rectæ cum alia ducitur per rectam poſitione datam; ſi eligantur quælibet tres, quatuor, aut quinque, e rectis K, L, M, N, P, Q, R, &c. quæ ſuis interſectionibus ductis per rectas poſitione datas connectuntur; hæ, duo, tria, aut quatuor puncta interſectionum habebunt, quæ datas poſitione percurrent; & ideo per Prop. 1, 21, 22. reliqua interſectionum puncta, in hiſce tribus, quatuor, aut quinque rectis, Sectiones Conicas deſcribent. Adeoque ſi fuerint tantum ſex rectæ K, L, M, N, P, Q, quæ ſeſe interſecent in 15 punctis, quorum quinque ducantur per rectas poſitione datas; quælibet tres, quatuor, aut quinque ex hiſce rectis ſeorſim conſiderari poſſunt, ad modum Prop. 1, 21, 22, ſumendo eas alternatim quot modis fieri licet, & proinde colligitur 10 puncta reliqua interſectionum rectarum K, L, M, N, P, Q, Sectiones Conicas deſcribere. Et ſimiliter ſi fuerint ſeptem rectæ K, L, M, N, P, Q, R, ſeſe interſecantes in 21 punctis, quorum ſex ducantur per rectas poſitione datas, quælibet tres, quatuor, quinque, aut ſex, ex iis ſeorſim conſiderari poſſunt, ſumendo eas alternatim quot modis fieri licet; & propterea 15 puncta reliqua interſectionum rectarum K, K, M, N, P, Q, R, Sectiones Conicas deſcribent. Et ita deinceps. Ergo ſi fuerint n rectæ K, L, M, N, P, Q, R, &c, ſeſe interſecantes in $\frac{n^2 - n}{2}$ punctis quorum $n-1$ puneta ducantur per rectas poſitione datas; quælibet tres, quatuor, quinque, ſex, ſeptem, octo, &c. ex iis ſeorſim conſiderari